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**THE PERILS OF FORWARD GUIDANCE**

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# Rational Inattention to News: The Perils of Forward Guidance

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**Résumé:** Cet article étudie la valeur sociale de l'information sur le future lorsque les agents sont rationnellement inattentifs. Dans un modèle à générations imbriquées d'inflation la banque centrale (BC) peut régler la masse monétaire en réponse au prix actuel. La BC a clairvoyance parfaite sur les T chocs futurs et dévoile cette information aux agents qui sont rationnellement inattentifs. A l'équilibre à anticipations rationnelles, les risques individuels et agrégés peuvent augmenter avec la diffusion de l'information lorsque le comportement monétaire n'est pas "assez réactif" et les agents ne sont "pas assez attentifs" aux nouvelles. En particulier, avec un horizon T plus long, les agents doivent être plus attentifs pour éviter des effets pervers sur le bien-être, alors que la notion de "assez réactif" reste invariante. En ce sens, une politique monétaire efficace est une condition préalable à une communication efficiente.

**Classification JEL:** E50, E58, E60, D83.

**Mots-clés:** Acquisition de l'Information, Communication de la Banque Centrale, Politique Monétaire, Valeur Sociale de l'Information.

**Abstract:** This paper studies the social value of information about the future when agents are rationally inattentive. In a stylized OLG model of inflation the central bank (CB) can set money supply in response to the current price. The CB has perfect foresight about the future $T$ shocks and releases this information to rationally inattentive agents. At the unique REE, individual and aggregate risks can increase with the release when the monetary conduct is not "tight enough" and agents are "not attentive enough" to the news. In particular, the shorter the $T$, the more attentive the agents must be to avoid perverse welfare effects, whereas the notion of "tight enough" remains invariant. In this sense, efficient communication requires effective monetary policy.

**JEL Classifications**: E50, E58, E60, D83.

**Keywords**: Information Acquisition, Central Bank Communication, Monetary Policy, Social Value of Information.

# 1 Introduction

Should a central bank (CB) disclose information about the future? Should the CB be concerned about how attentive the private sector is to the announcements? And could the answer depend on how limited is the CB in the use of its first-best conventional policy? These questions are motivated by the unfolding of the recent crisis which forced many CBs to develop non-conventional policies to overcome the ZLB of the interest rate. At the same time, the fear that uncertainty could become a dominant sentiment on the markets stimulated an increasing recourse to long-horizon communication. The effect of such a change in communication policy, referred to as forward guidance, has been empirically documented in a number of recent studies (Campbell and others, 2012; Del Negro and others, 2012; Kool and Thornton, 2012).

Forward guidance occurs typically in the form of an announcement on the joint combination of the expected future economic outlook and contingent policy actions[1]. Though the purpose of the monetary policy should be clear in principle, the informational content about the future economic course appears to be at least a matter of subjective interpretation. The innate problem of what the private sector understands about public announcements has always shed a shadow on the overall convenience of communication policies and forward guidance in particular.

Motivated by these issues this paper presents a theoretical analysis of the social value of information about the future. It emphasizes the perils of releasing information about the future, especially when its vagueness could exacerbate a difference in subjective beliefs and the CB is bound in the use of its first-best conventional policy. The key element is exploring the consequences of agents being rationally inattentive (a la Sims (2003)) to the central banker's announcements. This modeling tool captures the difficulty of the private sector to uniformly absorb available information which can arise due to both (or either) the communication transparency of the CB and the understanding of agents.

I use rational inattention to formalize the main idea that this paper has to offer to the debate. The release of information about the future entails a critical

[1]Cambell and others (2012) refer to these two aspects of forward guidance respectively as *Delphic* and *Odissean*. The latter refers to a public commitment by the CB to follow a certain path of future interest rates as suggested, for example, by Eggertsson and Woodford (2003) and Werning (2012). The former instead concerns a public release of the CB superior knowledge about the future which does not necessarily entail a commitment. This paper emphasizes the *Delphic* aspect of forward guidance although *Odissean* elements can be contextualized in two respects. First, in environments with informational asymmetries between the private sector and the CB an *Odissean* commitment cannot occur without an implicit *Delphic* release of information. Second, an *Odissean* commitment implies by definition a limitation in the ability of the CB to dampen aggregate fluctuations which is an element discussed in the paper.

trade-off. On the one hand, more information[2] allows agents to explain a larger fraction of macro-volatility. On the other hand, more information increases the volatility and persistence of macro outcomes as expectations become increasingly reactive to future shocks. Hence, the net effect is ambiguous: if agents are not "attentive enough" then the release of information can reduce - instead of increasing - their forecasting ability, and enhance - instead of dampening - the importance of aggregate fluctuations. In other words, the availability of dispersed information about the future can expose agents to *higher* - rather than lower - individual and aggregate risks. Importantly, this welfare perverse effect disappears if the sensitivity of the current outcome to the aggregate expectation is sufficiently low. This is where monetary policy can play a role.

To structure my study I present a simple OLG model of inflation[3] although the analysis is not tied to it. The results only rely on the forward-looking structure and the information setting considered. Agents live for two periods: each have one endowment for each period and makes a saving-consumption decision when young. The only saving technology is money whose aggregate quantity is subject to fundamental idiosyncratic shocks. The CB can eventually dampen aggregate price fluctuations implementing a price targeting policy. Although extremely simple, the model encapsulates the bulk of any dynamic forward-looking model: an aggregate outcome - the consumption price - depends linearly on the average expectation of the next outcome plus an exogenous disturbance. In this context I place the communication problem of interest. For each period I suppose that the CB has private information on the next $T$ shocks. The CB forms his own forecast about the future price and announces it to agents who are rationally inattentive. Agents receive the news in the form of a private signal whose precision is maximized in equilibrium under an informational capacity constraint. In turn, they form their forecast using the private signal and the information conveyed by the current price which is observed in the market.

I characterize the unique determinate rational expectation equilibrium for the full range of cases spanned by the time-horizon $T$, the agents' endowment of attention and the degree of tightness of the monetary policy. It turns out that if the monetary policy is not "tight enough" when agents are not "attentive enough", then the release of the CB's best inflation forecast can increase both individual and aggregate risks above the level obtained without the release. In particular the shorter the $T$, the more stringent the notion of "attentive enough", whereas the notion of "tight enough" remains invariant to the time horizon. In practice, if the CB, though limited in the use of its first-best policy, can still sufficiently

[2]In a rational inattention setup "more information" means both the availability of new information in front of an unchanged informational capacity, or an expansion of informational capacity in front of an unchanged availability of information.

[3]The model is similar to the ones proposed as a benchmark to study hyperinflation phenomena (Marcet and Nicolini, 2001; Sargent, Williams and Zha 2009 among others).

reduce price instability, the release of the information is always beneficial no matter how much attention the public pays. In this sense, the efficiency of the communication policy crucially depends on the effectiveness of the monetary conduct.

# 2 Related literature and methodological contribution

The idea that the anticipation of future shocks can explain expectation-driven business cycles has been explored by a recent literature on news (Beaudry and Portier (2006, 2007); Blanchard, L'Huiller and Lorenzoni (2009), Jaimovich and Rebelo (2009)). Typically, these works look at dynamic models where agents receive a public signal of a future shock. In contrast to the rational inattention approach agents do not have disparate beliefs about the future and their ability to forecast does not depend on the ongoing economic conditions.

Rational inattention has been used to explain a wide range of economic phenomena from cross-sectoral business-cycle facts (Sims 2006, Adam 2007, Mackowiak and Wienderholt 2009, Pasten 2012) to pricing behavior at the micro-level (Matejka 2011, Matejka and Sims 2010, Stevens 2012) and experimental data (Woodford 2012). Recent applications to communication policy include Chahour (2012) and Reis (2010) which are concerned, asthe present paper is, about suboptimal welfare outcomes when agents' attention to the CB announcements is limited. The latter focuses on what is the right timing to inform, whereas the former investigates the implications of endogenous provision and selection of information. In all these works, the object of attention is typically exogenous.

There are a number of studies on the perverse welfare effects of public and private information in static models with exogenous information structures. Notable examples are Morris and Shin (2002) and Angeletos and Pavan (2007). Part of these results are revisited in macro-models by Hellwig (2005) and Roca (2010). Colombo and others (2012) and Llosa and Venkateswaran (2012) are attempts to reconcile these findings with endogenous acquisition of information. Amador and Weill (2010) looks at the case when agents get endogenous information but cannot choose its precision.

To the best of my knowledge the present paper is the first one focusing on the macro effects of agents being rationally inattentive to an *endogenous* and *future* state with impact on current actions[4]. The endogeneity of the object of

[4]In Reis (2010) agents pay attention to a future exogenous regime shift which has no impact on today's decisions, but this information may increase their efficiency when the time of the regime change comes.

attention is at the origin of the general equilibrium failures documented by the main results of this paper. Moreover, I assume agents need to pay attention to get information about the future but do not need to pay attention to the current price from which they learn[5]. Such an hybrid informational setting implies that agents agree on the present and only can have disparate views about the future. In particular, the current price acts as a public signal which aggregates and partly reveals available information as in a Grossman and Stiglitz (1980) environment. The precision of public information therefore depends on the precision of the private signals which in turn are endogenous in a twofold sense: agents chose the precision of their private signal and the signal itself is a noisy observation of an endogenous future state.

# 3 A stylized OLG model of inflation

This section presents a simple micro-founded economy in which the policy maker could eventually exploit forward guidance to overcome the limits of the available conventional policy. The model will serve as a base for the more general analysis presented in the rest of the paper. Even if extremely stylized, it allows to naturally introduce and discuss a number of conceptual issues concerning the abstract task of modelling rational inattention to news. These are enlightened in a dedicated paragraph at the end of the section.

## 3.1 Basic setting

A continuum of agents indexed in the unit interval $I \equiv (0, 1)$ of generation $t > 1$ has available a two-period endowment of a unique perishable good $(w_0, w_1) = (2, 2w)$ where $w \in (0, 1)$. Preferences over consumption are given by

$$u\left(C_{i,t,0}, C_{i,t,1}\right) = \ln\left(C_{i,t,0}\right) + \ln\left(C_{i,t,1}\right)$$

subject to the budget constraints

$$C_{i,t,0} = 2 - \frac{M^d_{i,t}}{P_t} \quad \text{and} \quad C_{i,t,1} = 2w + \frac{M^d_{i,t}}{P_{t+1}} \tag{1}$$

where $C_i$ is individual consumption, $M^d_i$ is individual demand for money. The first subscript denotes the generation whereas the second one the periods in the agents' life. Agents of the initial generation at $t = 0$ live only one period, have preferences $u\left(C_{i,0,1}\right) = \ln C_{i,0,1}$ and they are endowed with $2w$ units of the consumption good.

[5]The effects of learning from prices have been recently emphasized by Amador and Weill (2010), Hellwig and Venkateswaran (2012) and Gaballo (2012).

Money is the only saving asset. The availability of money is subject to variations in time described by $\hat{u}_t \sim N(0,1)$. These can be interpreted as fundamental disturbances rather than accounting for policy actions taken by a central authority (i.e. positive or negative seigniorage[6]). The CB can tame price fluctuations implementing the following targeting rule

$$\frac{M_t^s}{P_t} = 1 - we^{-\hat{u}_t}\left(\frac{P_t}{\bar{P}}\right)^{\phi} \tag{2}$$

where $\phi \in (0,\infty)$ measures the tightness of the policy, and $\bar{P}$ is a fix price-level target. At the limit of $\phi \to 0$ the supply of real money is exogenously determined by $\hat{u}_t$, whereas with a (negative) positive $\phi$ the money supply (increases) decreases if the current price is above the price target, so that the price is stabilized. The first-best policy obtains at $\phi \to \infty$ for which prices are perfectly stabilized at the steady state level. Nevertheless, institutional rather than political constraints could prevent the monetary authority from achieving the first-best. In such a case the authority might be tempted to experiment alternative policies to boost welfare. The question I focus on is under which conditions information about the future can improve welfare for a given finite $\phi$. To this aim let me describe in detail what the authority knows and how communication can occur in this economy.

## 3.2 Information

I assume that at the beginning of time, Nature extracts the whole series of disturbances $\hat{\mathbf{u}}_t^{\infty}$ from $t = 0$ to $t \to \infty$. At each time $t$ the CB has private perfect foresight $\hat{\mathbf{u}}_t^{t+T}$ on the past, current and next $T$ monetary shocks where $T \in \{1, 2, ..., \infty\}$ indexes the horizon of the CB's perfect foresight ($T$-PF in short). The current price is instead public knowledge. At time $t$, the central banker forms her own price forecast

$$p_{t+1}^T \equiv \mathbf{E}[p_{t+1}|\hat{\mathbf{u}}_t^{t+T}] \tag{3}$$

where $p_{t+1} \equiv \left(P_{t+1} - \bar{P}\right)/\bar{P}$ denotes a deviation of the price from its deterministic steady state $\bar{P}$.[7] The forecast of the central bank represents a sufficient

[6]Notice that real money fluctuations yield both positive or negative seigniorage $\left(M_t^s - M_{t-1}^s\right)/P_t$. The fiscal-monetary authority is assumed to have an endowment sufficiently large to finance negative seigniorage. The endowment of the central authority can be constituted by lump sum taxes on the consumers' endowments.

[7]For notational purposes I anticipate here that the price target $\bar{P}$ represents both the deterministic and the stochastic steady state (see note 9). Notice that in equilibrium the price is log-normally distributed with steady state given by $P^* = P^m e^{\mathrm{V}(p)/2}$ where $P^m$ is the median of the price distribution which adjusts to satisfy $\bar{P} = P^m e^{\mathrm{V}(p)/2}$ for different values of the price variance $\mathrm{V}(p)$.

statistics of the best available information on the future price level. I will refer to such expectation as the $T$-perfect foresight ($T$-PF) price forecast.

In the attempt to provide forward guidance to agents the CB releases its private information to agents who cannot directly observe future shocks. Nonetheless agents' informational capacity is constrained, so they cannot fully absorb all the information conveyed. The announcement of the $T$-PF price forecast is received by agents in the form of a private noisy signal whose precision maximizes a constraint as usually assumed in the rational inattention literature. Formally, when young agent $i$ receives a private signal

$$\omega_{i,t} = p_{t+1}^{T} + \eta_{i,t} \tag{4}$$

where $\eta_{i,t}$ is an independent zero-centred disturbance whose variance $\sigma$ is endogenous to the optimization problem presented below. The limits of $\sigma \to 0$ and $\sigma \to \infty$ entail the cases of respectively perfect and null attentiveness to the CB announcements. Only at these two limits does informational heterogeneity vanish, otherwise informational asymmetries arise as agents suffer from different individual observational shocks. In particular the limit $\sigma \to \infty$ is an equivalent characterization of the no-information scenario where the CB does not release any information.

The distribution of the noisy signals are determined in equilibrium to satisfy

$$\mathrm{H}\left(p_{t+1}^{T}|p_t\right) - \mathrm{H}\left(p_{t+1}^{T}|p_t, \omega_{i,t}\right) \leq K \tag{5}$$

where $\mathrm{H}(\cdot) \equiv -\mathbf{E}\left[\ln f(\cdot)\right]$ is the usual entropy formula and $f(\cdot)$ is the probability density function. In equilibrium agents choose a distribution of signals such that the difference between the a-priori conditional entropy and the posterior conditional entropy does not exceed an exogenous parameter $K$. Notice that the object of attention $p_{t+1}^{T}$ is endogenous, that is, its distribution changes with $K$. This feature is at the core of the results of this paper.

## 3.3 Definition of an equilibrium

Once young agents get their private signals, they form an expectation about the future price

$$E_t^i p_{t+1} \equiv \mathbf{E}[p_{t+1}|p_t, \omega_{i,t}]$$

and demand money to save wealth accordingly. A definition of an equilibrium provides for market clearing with an optimal use of available attention. The formal statement follows.

**Definition 1** *For given parameters* $\{w, K, T, \phi, M_0, \bar{P}\}$, *a REE equilibrium is a series of prices and agents' expectations*

$$\{p_\tau, \{E_\tau^i p_{\tau+1}\}_I\}_{\tau=0}^{\infty}$$

*such that the money market clears and each individual makes a consumption-saving choice and an informational choice that maximize her own expected utility given constraints (1) and (5).*

Now we can derive optimal actions and so the course of the price. The optimal individual demand for money

$$\frac{M_{i,t}^{d}}{P_t} = 1 - w\frac{E_t^i P_{t+1}}{P_t} \tag{6}$$

depends on the individual forecast of the future price. In particular one can easily show that a second-order approximation of a expected *individual* utility loss is proportional to

$$\mathrm{V}\left(\Delta\right) = \int_0^1 \left(E_t^i p_{t+1} - p_{t+1}\right)^2 \mathrm{di} \tag{7}$$

being nothing else than the variance of the individual forecast error $\Delta_t \equiv E_t^i p_{t+1} - p_{t+1}$. This measure encapsulates the incentive of agents to pay attention to the authority's announcement: the more precise their price prediction, the more accurate their consumption-saving decision. A quadratic loss implies that the distribution of signals chosen by agents will be Gaussian[8] whose variance will be a function of $\kappa$.

To recap, agents make two choices when young. First, they choose the variance of the distribution of the signals such that their forecast error variance is minimized under the constraint (5). Second, once they form an expectation of the future price, they chose the quantity of money to bring in the second period maximizing expected consumption subject to the budget constraints (1). An equilibrium price is the price at which aggregate demand equals money supply $\int M_{i,t}^{d}\, di = M_t^s$.[9] The actual log-linear deviation[10] of the current price from the deterministic steady state is given by

$$p_t = \beta \bar{E}_t p_{t+1} + u_t \tag{8}$$

where and $\beta \equiv (1+\phi)^{-1}$, $u_t \equiv (1+\phi)^{-1}\hat{u}_t$ and $\bar{E}_t(\cdot) \equiv \int_0^1 E_t^i(\cdot)\,\mathrm{di}$ is the average market expectation expressed as the average of individual expectations. For the rest of the paper, the variance of $u_t$ will be denoted by $\sigma_u$ without any reference to the underlying $\sigma_{\hat{u}}$ as the results do not hinge on it.

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[8]Non-quadratic setting have been used to explain economic behavior at the micro level by Matejka (2011), Matejka and Sims (2010), Sims (2006), Stevens (2012) and Woodford (2011).

[9]In particular, from (2) and the average (6) we get

$$e^{-\hat{u}_t}\left(P_t/\bar{P}\right)^{\phi} = E_t P_{t+1}/P_t$$

which implies $P^* = \bar{P}$ along both the stochastic and deterministic steady state path characterized by $P^* = P_t = P_{t+1} = E_t P_{t+1}$ with different median price (see back note 5).

[10]The model could be equally transformed in logs without any approximations, but I prefer to proceed with log-linear prices to be perfectly consistent with the choice of a second-order welfare measure.

## 3.4 General insights from the model

Although highly stylized, the model just presented is general enough to characterizes the class of economies to which the results apply. In the following I remark three general features of the problem.

**A forward-looking reduced form.** The structural form (8) embodies the bulk of any forward-looking model. A current aggregate endogenous state, a price in our case, depends linearly from the average expectation of the future state plus an exogenous i.i.d. disturbance. This process is parametrized by a single coefficient $\phi$ which impacts on both the variance of the exogenous disturbances and the sensitivity of the current price to the average expectation. The latter, encapsulated by $\beta$, is an important element of our analysis whereas the former does not play any role as long as such variance is bounded and independent from the informational choice. In fact, we are interested in assessing - for a given $\beta$ - how changes in agents' information set impacts on different welfare dimensions measured in units of $\sigma_u$, the variance of the exogenous disturbances.[11]

**Availability of information about the future.** Rational inattention is a theory on available information about an uncertain state. In the model, available information about the future price is made available by the CB that truthfully releases a superior forecast about the future price. Nevertheless, one can remove this fiction and just assume that there is available information about the future $T$ shocks that agents can access investing some informational capacity. Appendix B clarifies that this is indeed an equivalent specification. In particular, I show there that the rational inattention approach is neutral with respect to the representation of the uncertain state. In other words, there are no gains in multiplying or disentangling the sources of information as long as agents have to pay attention to them subject to the same constraint.

**Learning from the current price.** Agents are capacity constrained on the acquisition of new information about the future, but not on the information revealed by the current price. These two pieces of information are intended to be different in nature. The noisy signals reflect the ability of each agent to read

[11] One can consider a different monetary policy

$$M_t^s/P_t = 1 - we^{-\hat{u}_t}\left(\hat{u}_t P_t/\bar{P}\right)^{\phi}$$

implying that the monetary action has no effect on the disturbance $\hat{u}_t$. This would yield a reduced form

$$p_t = \frac{1}{1-\phi}\bar{E}_t p_{t+1} + \hat{u}_t$$

where $\phi$ does not affect the variance of $\hat{u}_t$. None of the propositions presented in this paper would change. Nevertheless, I prefer the original specification (2) as it provides a more natural interpretation.

and understand the reports of the CB. On the contrary, the current price is not the result of any information search, but it is directly acquired through trading experience. This ensures that agents can only have disparate views about the future, but agree on the present.

# 4 Solving for the unique REE

I characterize the unique REE for the full range of cases spanned by the time-horizon $T$ of available information, the agents' attention endowment and the degree of tightness of the price-targeting policy. Notice that the content of this section is not tied to the specific model presented above, but instead relies on the three features just discussed. More specifically, the results are general in the class of models that can be mapped into a structural form like (8), an information setting described by (3)-(4)-(5) and a quadratic individual utility loss (7).

## 4.1 The actual law of motion

The information set of a young agent $i$ is composed by a normal *private* signal $\omega_{i,t} = \{p_{t+1}^T + \eta_{i,t}\}$ of the *T*-PF forecast whose variance has to be determined in equilibrium, and a *public* signal being the current price. Let us therefore fix a linear forecasting strategy weighing the current price and the signal about the future price[12]. Agent $i$ uses the following estimator for the mean of the future price realization

$$E_t^i p_{t+1} = a_i p_t + b_i \left(\beta^{-1} - a_i\right) \left(p_{t+1}^T + \eta_{i,t}\right) \tag{9}$$

where $a_i$ and $b_i$ are respectively the individual constant weights on the current price and the *private* signal multiplied for convenience and without loss of generality by a constant factor $\left(\beta^{-1} - a_i\right)$.

I firstly guess what is the form of the $p_t$ and then verify that this is consistent with (3), (8) and (9). Suppose the actual law of motion of the price is

$$p_t = \frac{1}{1 - \beta \mathbf{a}} \sum_{\tau=0}^{T} \mathbf{b}^\tau u_{t+\tau}, \tag{10}$$

where $\mathbf{b} \equiv \int b_i \, di$ and $\mathbf{a} \equiv \int a_i \, di$ are the average weights across the population. As a consequence the $T$-PF forecast has the form

$$p_{t+1}^T = p_{t+1} - \frac{1}{1 - \beta \mathbf{a}} \mathbf{b}^T u_{t+1+T}, \tag{11}$$

[12] A linear strategy is optimal when, as in this case, random variables involved in the signal extraction are normally distributed.

that is, the $T$-PF forecast generally deviates from the true price. In fact, information about $u_{t+1+T}$ is not available at time $t$, but it impacts the actual future price throughout the average price expectation formed at time $t+1$ when information about $u_{t+1+T}$ is indeed available. Notice that $\lim_{T\to\infty} p_{t+1}^{T} = p_{t+1}$ provided $|\mathbf{b}| < 1$; only at this limit do agents dirctly observe with a lag the announcement, which in this case corresponds to the actual price.

To prove that the guess is correct use (10) into (11) and substitute in the aggregate (9), which, once plugged back into (8), gives (10) again (derivation in appendix A.1.). This is true for any possible *disequilibrium* calibration of agents beliefs (9), that is, the guess holds without imposing rational expectation on agents' forecasts, but only on CB's ones.

Using (10) and (11), the current price can be expressed as a *public* signal of the $T$-PF forecast

$$p_t = \mathbf{b} p_{t+1}^{T} + \frac{1}{1-\beta\mathbf{a}} u_t \tag{12}$$

where notice corr$(p_{t+1}, u_t) = 0$. The precision of this signal about $p_{t+1}^{T}$ is given endogenously by $(\mathbf{b}(1-\beta\mathbf{a}))^2$. In particular, it decreases with the average weights $\mathbf{a}$ and $\mathbf{b}$ deviating away from respectively $\beta^{-1}$ and $0$. The current price does not fully reveal the $T$-PF given the presence of the current disturbance which blurs its informational content.

For any couple $(\mathbf{a}, \mathbf{b})$ such that $|\mathbf{b}| < 1$ and $\mathbf{a} \neq \beta^{-1}$ the price process is stationary with bounded variance

$$\sigma_p = \frac{1}{(1-\beta\mathbf{a})^2} \frac{1-\mathbf{b}^{2(T+1)}}{1-\mathbf{b}^2} \sigma_u, \tag{13}$$

and the a-priori volatility of the announcement is given by

$$\sigma_{p^T} = \frac{1}{(1-\beta\mathbf{a})^2} \frac{1-\mathbf{b}^{2T}}{1-\mathbf{b}^2} \sigma_u, \tag{14}$$

where the latter is strictly smaller than the former. The serial correlation of prices is given by $\mathbf{b}(1-\mathbf{b}^{2T})/(1-\mathbf{b}^{2(T+1)})$. A remark is in order here. In principle, (12) implies that only knowledge about $p_t$ and $u_t$ is actually needed to consistently predict $p_{t+1}^{T}$. Nevertheless, if (12) hold but (10)-(11) does not, the price process is a bubble embodying a non-fundamental component that grows at an exponential rate $\mathbf{b}^{-t}$. In other words, (12) describes all the infinite number of different expected price paths measurable with respect to current values $p_t$ and $u_t$, but only one among such paths - precisely the one for which (10)-(11) holds - is stationary with bounded variance $\sigma_p$. That means for the economy to be on the unique saddle-path necessarily the initial stock of money $M_0$ has to be set at a value $\bar{M}_0$ for which $p_0$ satisfies (10)-(11). I will assume this for the rest of the paper.

## 4.2 The equilibrium

The strategy to compute the equilibrium is in two steps. First, the precision of the private signal (the variance of the private observational error) is determined as a solution to the informational capacity problem. That is quite straightforward since all the signals are normally distributed. Given the solution to this problem, one can finally recover the restrictions on the profile of all individual weights $\{a_i,b_i\}_I$ imposing orthogonality conditions on the forecast errors.

Define $\kappa \equiv \left(e^{2K}-1\right)^{-1}$ an inverse measure of informational capacity, that is a measure of inattention. The following proposition states our first result.

**Proposition 2** *A unique REE stationary price process (10) and expectations paths (9) exist characterized by a profile of individual optimal average weights* $(a_i, b_i) = \left(\mathbf{a}_{(T)}, \mathbf{b}_{(T)}\right)$ *for each* $i$ *where*

$$\mathbf{a}_{(T)} = \frac{1-\mathbf{b}_{(T)}^{2T}}{1-\mathbf{b}_{(T)}^{2(T+1)}}\frac{\kappa}{1+\kappa}\mathbf{b}_{(T)},$$

*and* $\mathbf{b}_{(T)} \in (0,\beta)$ *is strictly decreasing in* $\kappa$ *and strictly increasing in* $T$ *and* $\beta$. *In particular, for given* $\kappa$ *and* $\beta$, $\mathbf{b}_{(T)}$ *is bounded above and below by respectively*

$$\mathbf{b}_{(\infty)} = \frac{1+\kappa-\sqrt{(1+\kappa)^2-4\kappa\beta^2}}{2\kappa\beta}, \tag{15}$$

*and*

$$\mathbf{b}_{(1)} = \frac{1}{9\Lambda}\left(9\Lambda^2+3\Lambda\beta+\beta^2-3\right) \tag{16}$$

*with*

$$\Lambda = \sqrt[3]{\frac{\beta}{3}-\frac{\kappa\beta}{2(1+\kappa)}+\frac{\beta^3}{27}+\sqrt{\left(\frac{\kappa\beta}{2(1+\kappa)}\right)^2-\frac{\kappa\beta^4}{27(1+\kappa)}-\frac{\kappa\beta^2}{3(1+\kappa)}+\frac{\left(1+\beta^2\right)^2}{27}}}.$$

*At the equilibrium the informational choice is* $\sigma = \kappa(1-\mathbf{b}_{(T)}^2)(1-\mathbf{b}_{(T)}^{2(T+1)})^{-1}\sigma_{p^T}$.

**Proof.** In appendix A.1.2. ■

Figure 1 plots the optimal weight $\mathbf{b}_{(1)}$ and $\mathbf{b}_{(\infty)}$ - respectively denoted by solid and dotted lines - as functions of $\kappa$. Four calibrations are considered: $\beta = 0.99$ in red, $\beta = 0.9$ in blue, $\beta = \sqrt{0.5}$ in brown, $\beta = 0.5$ in purple. For a given $\kappa$ and $\beta$, an equilibrium value $\mathbf{b}_{(T)}$ lies in between its upper and lower bounds $\mathbf{b}_{(1)}$ and $\mathbf{b}_{(\infty)}$; the locus of these points is denoted by a shadow area which, ceteris paribus, widens with $\beta$. At the limit $\kappa \to \infty$ - when the

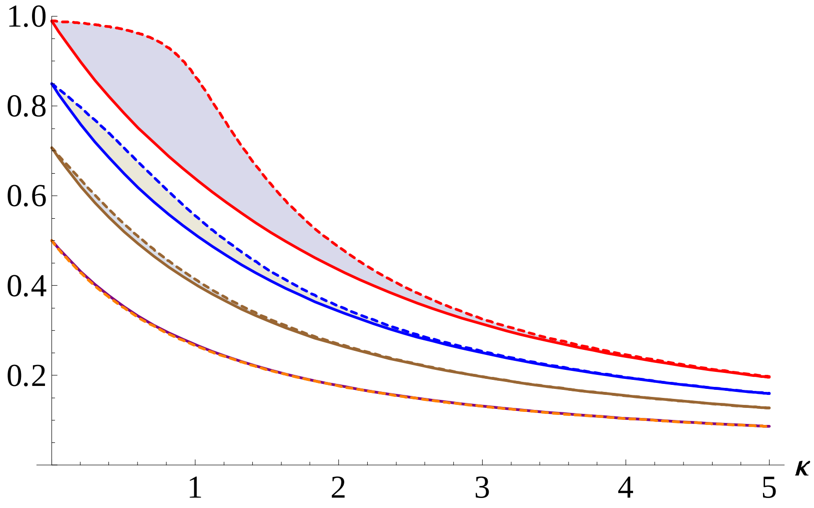


Figure 1: The average weight $\mathbf{b}$ as a function of the level of inattention $\kappa$, for $\beta = 0.99$ (red), $\beta = 0.85$ (blue), $\beta = \sqrt{0.5}$ (brown) and.$\beta = 0.5$ (purple). The dashed lines are obtained for $T \to \infty$ whereas solid ones for $T = 1$. Curves for finite values of $T$ lie in the encircled shadow areas.

private signal is not informative - then $(\mathbf{a}, \mathbf{b}) = (0, 0)$ is the only stationary solution. At the opposite limit $\kappa \to 0$ - when the signal is perfectly informative - $(\mathbf{a}, \mathbf{b}) = (0, \beta)$ is the only stationary solution (i.e. the current price is equal to the discounted sum of known future shocks). Ceteris paribus, the unique optimal weight $\mathbf{b}_{(T)}$ is monotonically increasing in $T$. That is, the shorter the horizon of available information, the less informative is the private signal; the higher is the difference between $p_t$ and $p_t^T$. For a given $T$ instead, as $\kappa$ increases or $\beta$ decreases, the precision of the private signal decreases and so the average weight $\mathbf{b}$ decreases too. The evolution of $\mathbf{a}$ is plotted in figure 2 which uses the same conventions. The current price noisily reveals aggregate information. When informational capacity is maximal the current price is not weighed as it is a redundant piece of information. As $\kappa$ increases above zero the current price is weighed as it provides information which refines the private understanding of the announcement. Nonetheless as $\kappa$ further increases private information becomes looser and so also the weight put on the current price must decrease.

## 4.3 Welfare dimensions

This sections proves that the availability of information about the future can generate welfare losses. I analyze two welfare components and their interaction. The first captures individual risk which increases with individual uncertainty about the future price. The second one concerns aggregate risk, that is, the dimension of aggregate fluctuations.

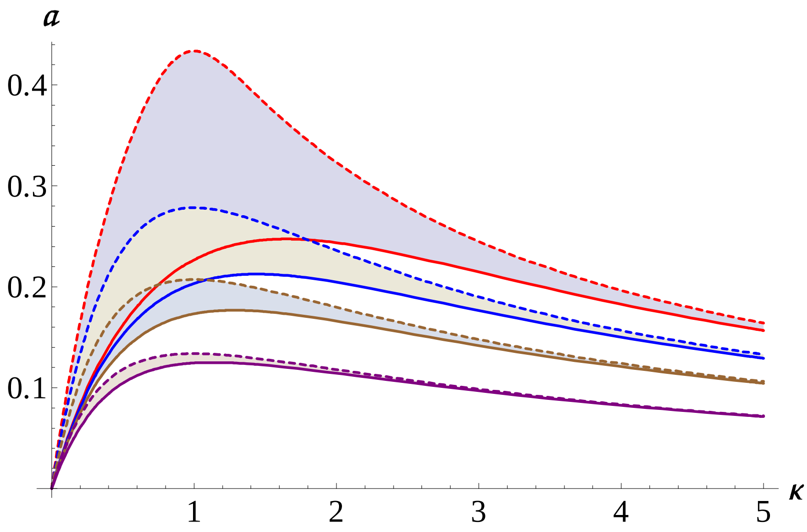


Figure 2: The average weight $\mathbf{a}$ as a function of the level of inattention $\kappa$, for $\beta = 0.99$ (red), $\beta = 0.85$ (blue), $\beta = \sqrt{0.5}$ (brown) and.$\beta = 0.5$ (purple). The dashed lines are obtained for $T \to \infty$ whereas solid ones for $T = 1$. Curves for finite values of $T$ lie in the encircled shadow areas.

### 4.3.1 Individual risk

Here I show that the availability of private information on the future price can in fact increase - instead of decreasing - the individual risk linked to the predicting ability of agents. This happens because more information makes agents able to explain a greater fraction of a larger price volatility, so that the overall effect is in fact ambiguous. According to (11), the forecast error variance depends on the accuracy of the information about the announcement and the horizon of the available information. The individual risk of making forecast errors is measured by the forecast error variance

$$\mathrm{V}(\Delta) = \mathrm{V}\left(p_{t+1}^{T} | p_t, \omega_{i,t}\right) + \frac{\mathbf{b}^{2T}}{\left(1-\beta\mathbf{a}\right)^2}\sigma_u, \tag{17}$$

where remember $\Delta_t \equiv E_t^i p_{t+1} - p_{t+1}$ denotes the individual forecasting mistake on the price at time $t+1$. It is composed by the conditional volatility of agents' forecast on the future price plus the volatility of the innovation which is unknown at the current time but will impact the next one. This latter component imposes a lower bound to the forecast error variance which decreases as $T$ increases. At the limit of $T \to \infty$, such lower bound is zero and the relation above collapses to $\mathrm{V}(\Delta) = \left(1 - \beta^{-1}\mathbf{b}_{(\infty)}\right)\sigma_u$, meaning that the individual utility loss is a fraction of the overall volatility.

In the transition between the perfect information $\kappa \to 0$ and no information scenario $\kappa \to \infty$, the course of $\mathrm{V}(\Delta)$ can be non-monotonic. In particular,

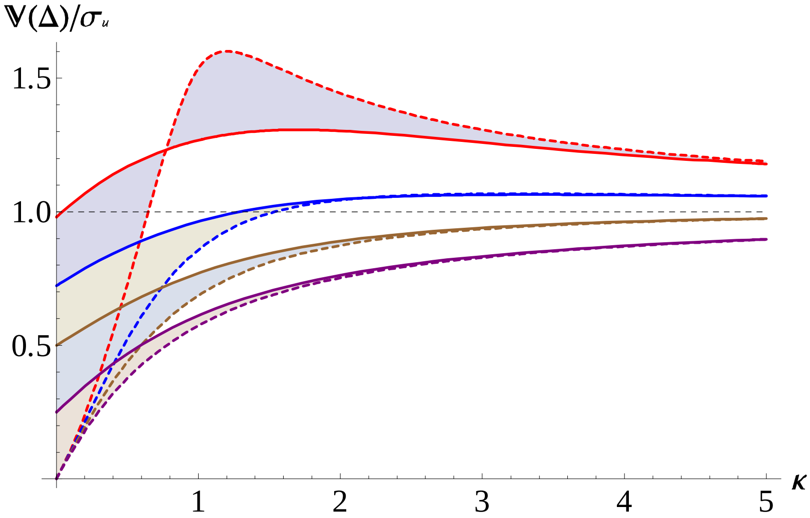


Figure 3: The forecast error variance $\mathrm{V}(\Delta)/\sigma_u$ as a function of the level of inattention $\kappa$, for $\beta = 0.99$ (red), $\beta = 0.85$ (blue), $\beta = \sqrt{0.5}$ (brown) and.$\beta = 0.5$ (purple). The dashed lines are obtained for $T \to \infty$ whereas solid ones for $T = 1$. Curves of finite values for $T$ lie in the encircled shadow areas.

for $\beta$ high enough and after a finite threshold value $\kappa^*$ of $\kappa$, the variance of the forecast error can remain above $\sigma_u$, namely its asymptotic value as $\kappa$ goes to infinity. The proposition below states the analytical result.

**Proposition 3** *There exists a threshold value $\kappa^*(T)$ decreasing in $\beta$ such that*

$$\mathrm{V}(\Delta)|_{\hat{\kappa},\beta} > \lim_{\kappa\to\infty} \mathrm{V}(\Delta)|_{\kappa,\beta} = \sigma_u \tag{18}$$

*for any $\hat{\kappa} > \kappa^*(T)$ if and only if $\beta > 1/\sqrt{2}$, whereas $\sup_\kappa\{\mathrm{V}(\Delta)|_{\kappa,\beta}\} = \sigma_u$ otherwise. Moreover $\kappa^*(T) < \kappa^*(T+1)$ for any finite $T$.*

**Proof.** Appendix A.2.3. ■

Figure 3 illustrates the proposition: it plots $\mathrm{V}(\Delta)$ as a function of $\kappa$ in units of $\sigma_u$ for the same calibrations and conventions of figure 1 and 2. For values of $\beta$ above $1/\sqrt{2}$ and for level of inattention above $\kappa^*$, the individual risk would be minimal in the no-information scenario. In other words, welfare improvements are guaranteed only if agents are sufficiently attentive so that the explanatory power of the information they receive is strong enough to gain on the increase in price volatility. Moreover, a shorter horizon $T$ makes a the release of information is inefficient at lower degrees of rational inattention. Such an effect does not obtain for values of $\beta$ below $1/\sqrt{2}$, that is when the current price does not react too much to the average price expectation. In this case the net effect of a release of information about the future is always positive.

### 4.3.2 Aggregate risk

Let us look now at inflation volatility. In the model, this is a source of aggregate risk to which agents of the same cohort are exposed as variations in seigniorage alters the distribution of real resources across generations. Notice that this component does not depend directly on the forecasting ability of agents, nevertheless it is affected by the amount of information absorbed by agents as a whole. The expression for the volatility of inflation in equilibrium is

$$\mathrm{V}(\pi) = (1-\mathbf{b})^2 \sigma_{p^T} + \frac{1+\mathbf{b}^{2T}}{(1-\beta\mathbf{a})^2}\sigma_u \tag{19}$$

which uses (11) and (12) with $\pi_t \equiv p_t - p_{t+1}$ denoting inflation. At the limit $\kappa \to 0$ the volatility of inflation is below $2\sigma_u$ whereas it measures $2\sigma_u$ as $\kappa$ goes to infinity. This is the effect of price correlation which reduces inflation. As for the forecast error variance the transition between the perfect information and no information scenario can be non-monotonic. The following proposition states an analytical result.

**Proposition 4** *There exists a threshold value $\kappa^{\#}(T)$ decreasing in $\beta$ such that*

$$\mathrm{V}(\pi)\,|_{\bar{\kappa},\beta} > \lim_{\kappa\to\infty} \mathrm{V}(\pi)\,|_{\kappa,\beta} = 2\sigma_u \tag{20}$$

*for any $\bar{\kappa} > \kappa^{\#}(T)$ if and only if $\beta > 1/2$, whereas $\sup_\kappa\{\mathrm{V}(\pi)\,|_{\kappa,\beta}\} = 2\sigma_u$ otherwise. Moreover $\kappa^{\#}(T) < \kappa^{\#}(T+1)$ for any finite $T$.*

**Proof.** Appendix A.1.4. ■

Figure 4 illustrates the proposition: it plots inflation volatility as a function of $\kappa$ in units of $\sigma_u$ with usual calibrations and conventions. For values of $\beta$ below $1/2$, the availability of information about the future price reduces inflation volatility in comparison to the no-information scenario. This is due to the fact that for high values of $\beta$, as $\kappa$ decreases, the serial correlation of prices can decrease less faster than price volatility. Nevertheless such an effect disappears for values of $\beta$ low enough, when current outcomes are less sensitive to the aggregate expectation which in fact embodies the correlated component of the price process.

### 4.3.3 Interaction of individual and aggregate risks

Finally, I am going to establish how individual and aggregate risks interact, that is, how the covariance between the two components evolves. This is stated by the following.

**Proposition 5** *The covariance between an aggregate fluctuation and an individual mistake*

$$\mathrm{cov}(\Delta\pi) = \mathrm{V}(\Delta) \tag{21}$$

*is equal to the variance of individual mistakes.*

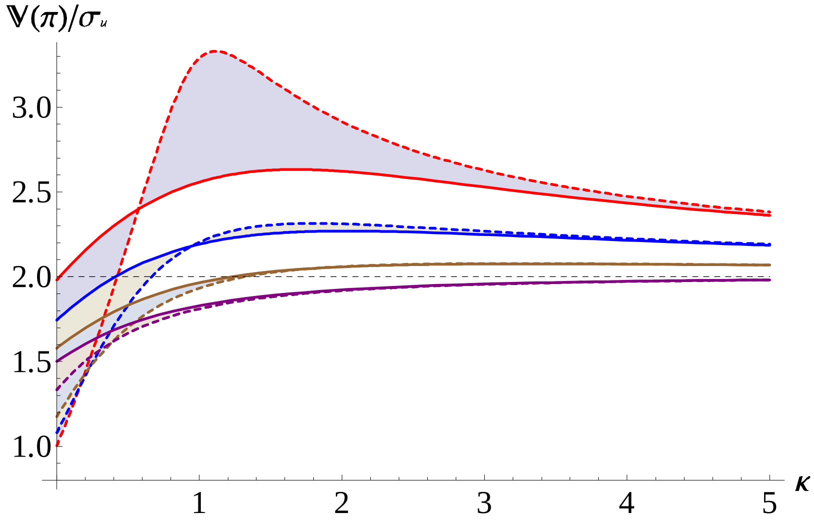


Figure 4: The inflation volatility $\mathrm{V}(\pi)/\sigma_u$ as a function of the level of inattention $\kappa$, for $\beta = 0.99$ (red), $\beta = 0.85$ (blue), $\beta = \sqrt{0.5}$ (brown) and.$\beta = 0.5$ (purple). The dashed lines are obtained for $T \to \infty$ whereas solid ones for $T = 1$. Curves for finite values for $T$ lie in the encircled shadow areas.

This result maintains that the covariance between aggregate and individual risks is positive, they co-move. Moreover, the covariance turns out to have the same evolution of the forecast error variance. Therefore proposition 3 equally applies to cov$(\Delta\pi)$.

# 5 Welfare in the stylized model

The findings above do not hinge on how the individual and aggregate risks impact into utility functions. In this sense these results go beyond the specific economy which I used as baseline. Nevertheless, how the two play into an overall welfare figure does depend on the details of the model. In particular, whenever the welfare contributions of individual and aggregate risks sum up to each other, the resulting welfare trivially inherits their qualitative features. Nevertheless, this is not the only possibility. The welfare contributions of the two risks can partly offset each other. The model presented in section 3 is able to shed light on this second possibility.

The welfare criterion I consider is the unconditional expected utility, namely $\mathbf{E}\left[u\left(C_{i,t,0}, C_{i,t,1}\right)\right]$. A second-order approximation of a welfare loss is obtained as

$$\mathrm{W} = \frac{1}{2}\left(\mathrm{V}\left(c_{i,t,1}\right) + \mathrm{V}\left(c_{i,t,2}\right)\right) \tag{22}$$

where

$$c_{i,t,1} = \frac{w}{1+w}\left(\Delta_t - \pi_t\right), \tag{23a}$$

$$c_{i,t,2} = \frac{1}{1+w}\pi_t - \frac{w}{1+w}\Delta_t, \tag{23b}$$

with $c_{i,t,\cdot} \equiv \left(C_{i,t,\cdot} - \bar{C}\right)/\bar{C}$ being a log-linear approximation of individual consumption around the deterministic steady state $\bar{C}$. Notice that, ceteris paribus, a higher expected inflation increase consumption when young and decrease expected consumption when hold. In particular, an overestimation of the future price level (a positive forecasting mistake $\Delta_t$) impacts positively on the consumption of agents when young and negatively on the consumption when old, whereas an actual deflation (a positive $\pi_t$) has opposite effects. We can rewrite the welfare loss as

$$\mathrm{W} = \frac{1+w^2}{2\left(1+w\right)^2}\mathrm{V}\left(\pi\right) - \frac{w}{\left(1+w\right)^2}\mathrm{V}\left(\Delta\right). \tag{24}$$

after using (21) (details in appendix A.6). It is quite easy to compare welfare losses in the cases agents have respectively no information ($\kappa \to \infty$) and all the available information ($\kappa \to 0$). In the first case agents have no informational capacity and/or the authority does not make the announcement, whereas in the second agents have infinite informational capacity and the authority makes the announcement. The following proposition holds.

**Remark 6** *For given $w$ and $\beta$, the welfare loss at the no-information equilibrium ($\kappa \to \infty$) is strictly higher than the welfare loss at the fully attentive equilibrium ($\kappa \to 0$) if and only if*

$$\lim_{\kappa\to 0}\mathrm{W} = \frac{1+w^2}{\left(1+w\right)^2}B < \lim_{\kappa\to\infty}\mathrm{W} = \frac{w^2+1-w}{\left(1+w\right)^2}$$

*with $B \equiv \left(1+\beta^{2T+1}\right)/\left(1+\beta\right) < 1$, that is when*

$$w < \frac{1-\sqrt{1-4\left(1-B\right)^2}}{2\left(1-B\right)}.$$

Depending on $w$, the model can account for two opposite welfare outcomes at the fully attentive equilibrium. One in which perfect anticipation of the shocks would improve welfare - i.e. perfectly knowing the shocks would be "good" - and an other in which instead this would decrease welfare - i.e. perfectly knowing the shocks would be "bad". In both cases agents have individual incentive to acquire the information as this improves their forecast, but in the latter the

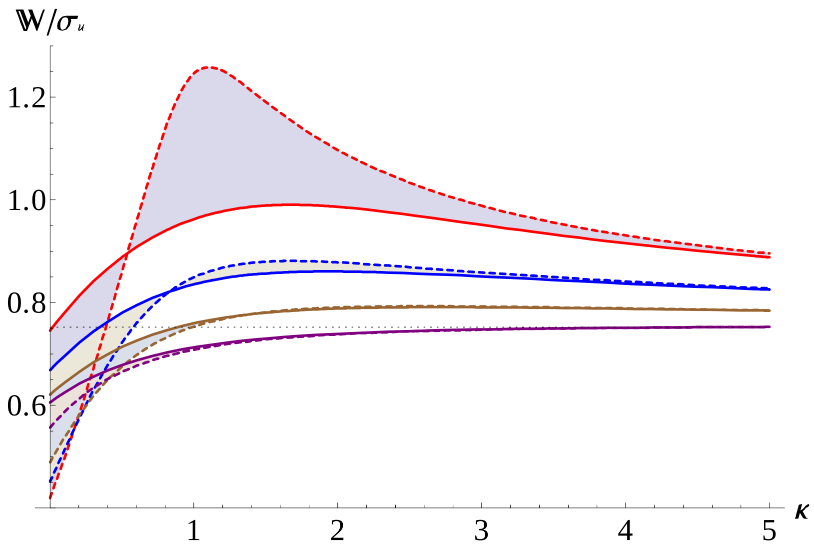


Figure 5: The welfare loss $\mathrm{W}/\sigma_u$ as a function of the level of inattention $\kappa$, for $w = 0.9$, and $\beta = 0.99$ (red), $\beta = 0.85$ (blue), $\beta = \sqrt{0.5}$ (brown) and.$\beta = 0.5$ (purple). The dashed lines are obtained for $T \to \infty$ whereas solid ones for $T = 1$. Curves for finite values for $T$ lie in the encircled shadow areas.

shocks are a source of general equilibrium inefficiency. This result obtains because at the fully attentive scenario the forecast error is null and does not counteract the welfare loss deriving from inflation. The stabilizing effect, which arises as the variance of the forecast error increases, is stronger with a larger $w$: the higher the real endowment in the second period, the more reactive the individual demand for money to expected inflation.

This possibility clarifies that the non-monotone welfare effects which I have documented are independent from the nature of the impact of the shocks at the fully attentive equilibrium. In the most interesting case, the social efficiency of the release of information about the future can be overturned for low enough levels of attention. In the other case instead the release of information remains inefficient for whatever level of rational inattention. The two cases are illustrated by figures 5 and 6 where the overall welfare is plotted as a function of $\kappa$ for respectively $w = 0.1$ and $w = 0.9$ with the same conventions of previous pictures. In the first graph, all curves originate from points below the no-information outcome at 0.75. In the second instead all curves start from points above the no-information outcome at about 0.255. In both, a non-linearity shows up which is stronger with a higher $\beta$: welfare loss is initially rising in $\kappa$ and then decreases after a threshold towards the no-information value.

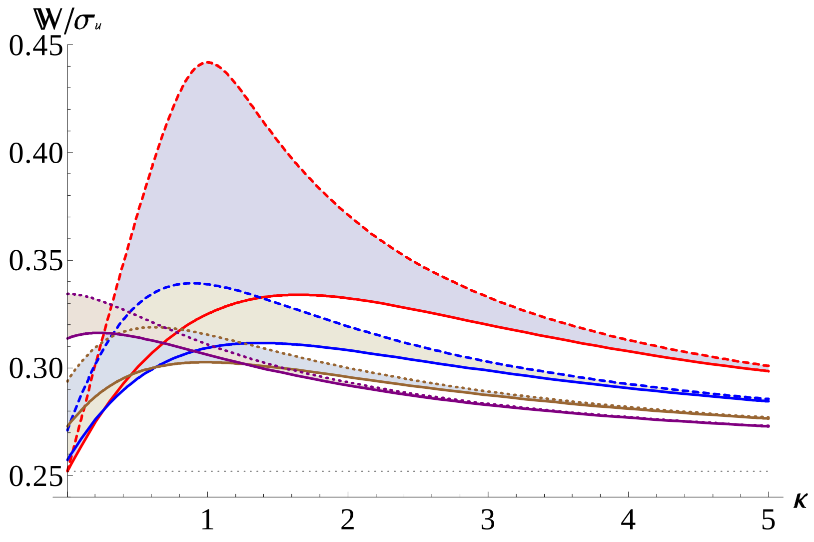


Figure 6: The welfare loss $\mathrm{W}/\sigma_u$ as a function of the level of inattention $\kappa$, for $w = 0.9$, and $\beta = 0.99$ (red), $\beta = 0.85$ (blue), $\beta = \sqrt{0.5}$ (brown) and.$\beta = 0.5$ (purple). The dashed lines are obtained for $T \to \infty$ whereas solid ones for $T = 1$. Curves for finite values for $T$ lie in the encircled shadow areas.

# 6 Concluding remarks on possible extensions

This paper formalizes a trade-off in the release of information about the future when agents are rationally inattentive: more information increases the fraction of macro-volatility that agents can explain, but also increases macro-volatility itself. The efficiency of communication relies therefore on the relative sensitivity of these two dimensions to information. Whenever the central authority has the instruments to sufficiently lower the reaction of current outcomes to expectations, then the release of socially beneficial information remains always such. Otherwise, it exists a threshold of attention below which the release can decrease - instead of increasing - both individual and aggregate risks. In this sense, efficient communication requires sufficiently tight monetary policy. To conclude, let me discuss the possibility to extend the setting along different directions.

In the model agents receive information during the only period where they are active, when young. This is a feature of OLG models which allows to abstract from memory. One could wonder how the results change assuming long-living agents who can potentially collect different private signals, one for each period of activity, about correlated future states. Two points are worth to be remarked in this respect. First, this assumption does not affect the results in the benchmark case with $T \to \infty$. With an infinite horizon in fact $\lim_{T\to\infty} p^T_{t+1} = p_{t+1}$, that is, the CB announces exactly the future price which is perfectly observed by agents next period, therefore memory is irrelevant. Second, as already mentioned, rational inattention is neutral with respect to the representation of the uncertain state no matter how many sources of information agents have available. This argument would equally apply to past information if we imagine that agents need to pay attention to past data. Also in this case results would not change. Nevertheless at the moment there not exists a fully-fledged theory of how to incorporate memory into the rational inattention approach. This very delicate issue is out of the scope of this paper and it deserves a full focus which is left for future research.

Finally, what would happen if agents could pay a real cost to enlarge their informational capacity? This extension concerns how the acquisition of informational capacity would endogenously impact agents' utility in the specific model. Although interesting, this extension would not affect the main results which are independent from the details of the utility functions. Agents would acquire the amount of capacity for which the cost of one more unit equates the marginal gain in utility. Therefore any quantity of capacity can result in equilibrium for an appropriate value of the cost. In this respect, taking an exogenous information capacity or an exogenous cost of informational capacity would have been equivalent, although less transparent, for the aim of this study.

# Appendix A: derivations and proofs

**A.1. Check of the guess (pag. 11)** Using (10) into (11) and substituting into the average (9) we have

$$E_t p_{t+1} = \mathbf{a} p_t + \mathbf{b}\left(\beta^{-1} - \mathbf{a}\right)\left(\frac{1}{1-\beta\mathbf{a}}\sum_{\tau=0}^{T-1}\mathbf{b}^{\tau} u_{t+1+\tau}\right),$$

that plugged into (8) gives

$$p_t = \beta\mathbf{a} p_t + \sum_{\tau=0}^{T-1}\mathbf{b}^{\tau+1} u_{t+1+\tau} + u_t,$$

and finally

$$p_t = \frac{1}{1-\beta\mathbf{a}}\left(\sum_{\tau=0}^{T-1}\mathbf{b}^{\tau+1} u_{t+1+\tau} + u_t\right) = \frac{1}{1-\beta\mathbf{a}}\sum_{\tau=0}^{T}\mathbf{b}^{\tau} u_{t+\tau},$$

which is (10) again. The guess is hence verified for any couple $(\mathbf{a}, \mathbf{b})$.

**A.2. Proof of proposition 2** *First step: the informational choice.* Since both signals are normally distributed, the capacity constraint is

$$\mathrm{H}\left(p_{t+1}^{T}|p_t\right) - \mathrm{H}\left(p_{t+1}^{T}|\, p_t,\omega_{i,t}\right) = \frac{1}{2}\log\left(\frac{\mathrm{V}\left(p_{t+1}^{T}|p_t\right)}{\mathrm{V}\left(p_{t+1}^{T}|p_t,\omega_{i,t}\right)}\right) \leq K, \tag{25}$$

where the ex-ante and ex-post conditional volatility are easily obtained as

$$\mathrm{V}\left(p_{t+1}^{T}|p_t\right) = \left(\frac{1}{\sigma_{p^T}} + \frac{\mathbf{b}^2\left(1-\mathbf{b}^{2T}\right)}{\left(1-\mathbf{b}^2\right)\sigma_{p^T}}\right)^{-1} = \frac{\left(1-\mathbf{b}^2\right)\sigma_{p^T}}{\left(1-\mathbf{b}^{2(T+1)}\right)},$$

and

$$\mathrm{V}\left(p_{t+1}^{T}|\omega_{i,t},p_t\right) = \left(\frac{1}{\sigma_{p^T}} + \frac{\mathbf{b}^2\left(1-\mathbf{b}^{2T}\right)}{\left(1-\mathbf{b}^2\right)\sigma_{p^T}} + \frac{1}{\sigma}\right)^{-1} = \frac{\left(1-\mathbf{b}^2\right)\sigma_{p^T}\sigma}{\left(1-\mathbf{b}^2\right)\sigma_{p^T} + \left(1-\mathbf{b}^{2(T+1)}\right)\sigma} \tag{26}$$

where the term $\left(1-\mathbf{b}^{2T}\right)\mathbf{b}^2/\left(1-\mathbf{b}^2\right)\sigma_{p^T}$ measures the precision of the information conveyed by the current price obtained according to (12) and (14). In equilibrium we get

$$\sigma = \frac{\kappa\left(1-\mathbf{b}^2\right)}{1-\mathbf{b}^{2(T+1)}}\sigma_{p^T} \tag{27}$$

with $\kappa \equiv \left(e^{2K}-1\right)^{-1}$.

*Second step: optimal weights.* The optimal weights are given by the orthogonality restrictions

$$p_t : \mathbf{b}\sigma_{p^T} - \mathbf{a}\frac{1-\mathbf{b}^{2(T+1)}}{1-\mathbf{b}^{2T}}\sigma_{p^T} - \left(\beta^{-1} - \mathbf{a}\right)\mathbf{b}^2\sigma_{p^T} = 0, \tag{28}$$

and

$$p_{t+1}^T + \eta_{i,t} : \sigma_{p^T} - \mathbf{a}\mathbf{b}\sigma_{p^T} - \left(\beta^{-1} - \mathbf{a}\right)\mathbf{b}\left(\sigma_{p^T} + \sigma\right) = 0, \tag{29}$$

where I used the relations $E(p_t p_{t+1}^T) = \mathbf{b}\sigma_{p^T}$ and $\sigma_p = \sigma_{p^T}\left(1-\mathbf{b}^{2(T+1)}\right)/\left(1-\mathbf{b}^{2T}\right)$. From the first equation we get

$$\mathbf{a} = \frac{\mathbf{b}\left(1-\beta^{-1}\mathbf{b}\right)}{(1-\mathbf{b}^2)}\left(1-\mathbf{b}^{2T}\right), \tag{30}$$

that plugged into the second jointly with (27) yields

$$\frac{\kappa}{1+\kappa} = \underbrace{\frac{\left(1-\beta^{-1}\mathbf{b}\right)}{(1-\mathbf{b}^2)}}_{f(\mathbf{b})}\underbrace{\left(1-\mathbf{b}^{2(T+1)}\right)}_{g(\mathbf{b},T)} \equiv \Phi(\mathbf{b},T) \tag{31}$$

which characterizes equilibrium values for $\mathbf{b}$. We are looking for a stable solution, that is for equilibrium values $|\mathbf{b}| < 1$ for which $\lim_{T\to\infty}\mathbf{b}^{2(T+1)} = 0$. At the limit $T \to \infty$ the relation above gives (15) as the unique stable solution. Notice further that, for given $\beta$ and $T$, it is $\partial\Phi/\partial\mathbf{b} < 0$ since $\partial f/\partial\mathbf{b} < 0$, $f > 0$ and $\partial g/\partial\mathbf{b} < 0$, $g > 0$ in the all range $(0,1)$. Therefore for a given $T$ and $\beta$, there exists a unique solution $\mathbf{b}$ which is strictly decreasing in $\kappa$. Moreover, for a given $\kappa$ and $T$ the equilibrium $\mathbf{b}$ value has to be increasing in $\beta$ given that $f$, and so $\Phi$, is strictly increasing in $\beta$. Finally since $\partial\Phi/\partial T > 0$ we can conclude that for a given $\kappa$ and $\beta$ the equilibrium $\mathbf{b}$ is also strictly increasing in $T$. In particular at the lower bound $T = 1$ we have $\kappa/(1+\kappa) = \left(1-\beta^{-1}\mathbf{b}_{(1)}\right)(1+\mathbf{b}_{(1)}^2)$ whose unique real solution is (16) representing a lower bound for all equilibrium values of $\mathbf{b}$, for given $\kappa$ and $\beta$.

**A.3 Proof. of proposition 3** First let us recover an analytical expression for the forecast error variance measured in units of $\sigma_u$. We can rewrite (17) by plugging (26), (14) and (27) into the first term, (30) and (14) into second, and then using (31) to get rid of $\kappa$. We obtain

$$\frac{\mathrm{V}(\Delta)}{\sigma_u} = \left(1-\mathbf{b}^2\right)\frac{1-\mathbf{b}\beta^{-1} + \mathbf{b}^{2T+1}\left(\beta^{-1}-\mathbf{b}\right)}{\left(1-\mathbf{b}\beta + \mathbf{b}^{2T+1}(\beta-\mathbf{b})\right)^2}, \tag{32}$$

which is a function in $\mathbf{b}$, $\beta$ and $T$. We already proved that $\mathbf{b}$ is a strictly monotonic function of $\kappa$ ranging in $(0,\beta)$ for given $\beta$ and $T$. We use this fact to look for the conditions under which $\mathrm{V}(\Delta)/\sigma_u > 1$ for some $\kappa$. This inequality is equivalent to the following one

$$\Gamma(\mathbf{b},\beta,T) \equiv \beta\mathbf{b}\left(1-\mathbf{b}^{2(T+1)}\right)+\mathbf{b}\beta^3\left(1-\mathbf{b}^{2T}\right)-2\beta^2\left(1-\mathbf{b}^{2(T+1)}\right)+1-\mathbf{b}^2 < 0$$

which reduces to

$$\Gamma(\mathbf{b},\beta,\infty) = \beta\mathbf{b} + \mathbf{b}\beta^3 - 2\beta^2 + 1 - \mathbf{b}^2 < 0$$

at the limit of an infinite horizon. For the latter to hold it must be

$$\mathbf{b} < \ q_- \equiv \frac{\beta + \beta^3 - \left(1-\beta^2\right)\sqrt{\beta^2+4}}{2},$$

where notice $q_+ > 1 > \mathbf{b}$. Since $\mathbf{b}$ is monotonically decreasing in $\kappa$, then the inequality is satisfied for values of $\hat{\kappa}$ larger than some threshold $\kappa^*(\infty)$ provided $q_-$ is positive, that is, with $\beta > \sqrt{1/2}$.

To assess $\Gamma(\mathbf{b},\beta,T) < 0$ for finite values of $T$ notice that $\Gamma(\mathbf{b},\beta,T) - \Gamma(\mathbf{b},\beta,\infty) = -\mathbf{b}^{2T+1}\beta\left(\mathbf{b}-\beta\right)^2$ is always negative. This means that, for given $\mathbf{b}$ and $\beta$, if $\Gamma(\mathbf{b},\beta,\infty) < 0$ then also $\Gamma(\mathbf{b},\beta,T) < 0$. Therefore we know that $\beta > \sqrt{1/2}$ is a sufficient condition for $\Gamma(\mathbf{b},\beta,T) < 0$ for some $\mathbf{b}$ small enough - meaning for any $\hat{\kappa}$ larger than some threshold $\kappa^*(T)$. Moreover since $\Gamma(\mathbf{b},\beta,T) < \Gamma(\mathbf{b},\beta,T+1)$ it must be $\kappa^*(T) > \kappa^*(T+1)$ for a finite $T$.

To show that $\beta > \sqrt{1/2}$ is also a necessary condition to obtain $\Gamma(\mathbf{b},\beta,T) < 0$ for any $\mathbf{b}$ small enough it is enough to note that at any $T$ it must be

$$\lim_{\mathbf{b}\to 0}\Gamma(\mathbf{b},\beta,T) = -2\beta^2 + 1 < 0$$

which requires $\beta > \sqrt{1/2}$.

**A.4. Proof. of proposition 4** Using (14) and (30) we can rewrite (19) as

$$\frac{\mathrm{V}\left(\pi\right)}{\sigma_u} = 2\left(1-\mathbf{b}^2\right)\frac{\left(1+\mathbf{b}^{2T+1}\right)\left(1-\mathbf{b}\right)}{\left(1-\mathbf{b}\beta+\mathbf{b}^{2T+1}\left(\beta-\mathbf{b}\right)\right)^2},$$

which is a function in $\mathbf{b}$, $\beta$ and $T$. We already proved that $\mathbf{b}$ is a strictly monotonic function of $\kappa$ ranging in $(0,\beta)$ for given $\beta$ and $T$. We use this fact to look for the conditions under which $\mathrm{V}\left(\pi\right)/\sigma_u > 2$ for some $\kappa$. This inequality is equivalent to the following one

$$\Theta(\mathbf{b},\beta,T) \equiv \left(1-\mathbf{b}^{2T}\right)\mathbf{b}\beta^2 - 2\left(1-\mathbf{b}^{2(T+1)}\right)\beta + \mathbf{b}\left(1-\mathbf{b}^{2(T+1)}\right) + 1 - \mathbf{b}^2 < 0$$

which reduces to

$$\Theta(\mathbf{b},\beta,\infty) = \mathbf{b}\beta^2 - 2\beta + \mathbf{b} + 1 - \mathbf{b}^2 < 0$$

at the limit of an infinite horizon. For the latter to hold it must be

$$\mathbf{b} < \ s_{-} \equiv \frac{1+\beta^2-(1-\beta)\sqrt{\beta^2+2\beta+5}}{2}$$

where notice $s_{+} > 1 > \mathbf{b}$. Since $\mathbf{b}$ is monotonically decreasing in $\kappa$, then the inequality is satisfied for values of $\bar{\kappa}$ larger than some threshold $\kappa^{\#}(\infty)$ provided $s_{-}$ is positive, that is, with $\beta > 1/2$.

To assess the inequality for finite values of $T$ notice that $\Theta(\mathbf{b},\beta,T)-\Theta(\mathbf{b},\beta,\infty) = -\mathbf{b}^{2T+1}(\mathbf{b}-\beta)^2$ is always negative. This means that, for given $\mathbf{b}$ and $\beta$, if $\Theta(\mathbf{b},\beta,\infty) < 0$ then also $\Theta(\mathbf{b},\beta,T) < 0$. Therefore we know that $\beta > 1/2$ is a sufficient condition for $\Theta(\mathbf{b},\beta,T) < 0$ for some $\mathbf{b}$ small enough - meaning for any $\bar{\kappa}$ larger than some threshold $\kappa^{\#}(T)$. Moreover since $\Theta(\mathbf{b},\beta,T) < \Theta(\mathbf{b},\beta,T+1)$ it must be $\kappa^{*}(T) > \kappa^{*}(T+1)$ for a finite $T$.

To show that $\beta > 1/2$ is also a necessary condition to obtain $\Theta(\mathbf{b},\beta,T) < 0$ for any $\mathbf{b}$ small enough it is enough to note that at any $T$ it must be

$$\lim_{\mathbf{b}\to 0}\Theta(\mathbf{b},\beta,T) = -2\beta+1 < 0$$

which requires $\beta > 1/2$.

**A.5. Derivation of the correlation between welfare components.** Using (9), (11), (12) and finally (30) we can rewrite the covariance as

$$\frac{\text{cov}\,(\Delta\pi)}{\sigma_u} = \mathbf{E}\left(\frac{\mathbf{a}}{1-\beta\mathbf{a}}u_t + \left(\beta^{-1}\mathbf{b}-1\right)p_{t+1}^T + \mathbf{b}\left(\beta^{-1}-\mathbf{a}\right)\eta_i - \frac{\mathbf{b}^T}{1-\beta\mathbf{a}}u_{t+1+T}\right)$$
$$\left(\frac{1}{1-\beta\mathbf{a}}u_t + (\mathbf{b}-1)\,p_{t+1}^T - \frac{\mathbf{b}^T}{1-\beta\mathbf{a}}u_{t+1+T}\right)\sigma_u^{-1} =$$

$$= \frac{\mathbf{b}\left(1-\beta^{-1}\mathbf{b}\right)\left(1-\mathbf{b}^{2T}\right)+\left(1-\beta^{-1}\mathbf{b}\right)(1-\mathbf{b})\left(1-\mathbf{b}^{2T}\right)+\left(1-\mathbf{b}^2\right)\mathbf{b}^{2T}}{\left(1-\mathbf{b}^2\right)(1-\beta\mathbf{a})^2} =$$

$$= \frac{1-\beta^{-1}\mathbf{b}+\mathbf{b}^{2T+1}\left(\beta^{-1}-\mathbf{b}\right)}{\left(1-\mathbf{b}^2\right)(1-\beta\mathbf{a})^2}.$$

To show that this is equivalent to the expression for the variance of the forecast error it is enough to confront the expression above with (32) and notice that

$$\left(1-\mathbf{b}^2\right)(1-\beta\mathbf{a}) = \left(1-\mathbf{b}\beta+\mathbf{b}^{2T+1}(\beta-\mathbf{b})\right)$$

can be easily proved using (30).

**A.6. Derivation of the Welfare** Using (6) and (1) the level of individual consumption can be expressed as functions of the individual price expectation and the current and future price as follows

$$\begin{aligned} C_{i,t,1} &= 1 + w\frac{E_t^i P_{t+1}}{P_t}, \\ C_{i,t,2} &= 2w + \frac{P_t}{P_{t+1}} - w\frac{E_t^i P_{t+1}}{P_{t+1}}. \end{aligned}$$

whose linearization around the steady state gives (23). A second-order approximation of aggregate welfare from the steady state is given by (22) where

$$\begin{aligned} \mathrm{V}\left(c_{i,t,1}\right) &= \frac{w^2}{\left(1+w\right)^2}\mathrm{V}\left(\Delta\right) + \frac{w^2}{\left(1+w\right)^2}\mathrm{V}\left(\pi\right) - \frac{2w^2}{\left(1+w\right)^2}\mathrm{cov}\left(\Delta\pi\right), \\ \mathrm{V}\left(c_{i,t,2}\right) &= \frac{w^2}{\left(1+w\right)^2}\mathrm{V}\left(\Delta\right) + \frac{1}{\left(1+w\right)^2}\mathrm{V}\left(\pi\right) - \frac{2w}{\left(1+w\right)^2}\mathrm{cov}\left(\Delta\pi\right), \end{aligned}$$

whose sum gives (24) after plugging (21) inside.

# Appendix B: Equivalent specifications

In this section I will prove that the equilibrium characterized above does not depend on the fact that agents receive a private signal of the CB price forecast rather than private signals about the single future shocks. To keep the exposition concise I will limit the discussion to the case $T \to \infty$. This result obtains since the entropy constraint is neutral with respect to the specification of the state. In other words it does not matter whether agents receive a private noise of the single shock or a private signal of a linear combination of them. What matters is to which extent this information reduces the uncertainty on the best available information on the future, that is the $\infty$-PF forecast.

## B.1. Announcing the $\infty$-PF inflation forecast and the current money shock

Information about the current monetary shock would be needed to increase the informativeness of the current price about the future one. Nevertheless, given the entropy constraint, the more signals are released the less precise will be their reception. In this section I will develop the case when agents receive two private signals: one about the $\infty$-PF price forecast as before, $p_{t+1}^{\infty} + \eta_{i,t}$, and the other about the current monetary shock, $u_t + \eta_{i,t|t}$, where, for current and future reference, I define $\eta_{i,t|\tau} \sim N\left(0, \sigma_{(\tau)}\right)$ as a private disturbance on the observation at time $t$ of the monetary shock at time $\tau$ whose variance $\sigma_{(\tau)}$ has now to be determined jointly to $\sigma$ to maximize (5).

As before, we start from the guess that $p_{t+1}^{\infty} = p_{t+1}$ which will be verified at the end. I fix a linear forecasting strategy weighting the current price and the two private signals

$$E_t^i p_{t+1} = \hat{a}_i p_t + \hat{b}_i \left(\beta^{-1} - \hat{a}_i\right) \left(p_{t+1} + \eta_{i,t}\right) + \hat{c}_i \left(u_t + \eta_{i,t|t}\right) \tag{33}$$

where $(\hat{a}_i, \hat{b}_i)$ play the same role as in the analysis above and $\hat{c}_i$ is now an individual weight put on the additional private signal on the current monetary shock. In analogy to the previous steps, one can substitutes for (8) and iterate to obtain the actual law of prices as

$$p_t = \frac{1 + \beta \hat{\mathbf{c}}}{1 - \beta \hat{\mathbf{a}}} \sum_{\tau=0}^{\infty} \hat{\mathbf{b}}^{\tau} u_{t+\tau} \tag{34}$$

where bold weights denote average weights. The price process has bounded variance

$$\sigma_p = \frac{\left(1 - \beta \hat{\mathbf{c}}\right)^2}{\left(1 - \beta \hat{\mathbf{a}}\right)^2 \left(1 - \hat{\mathbf{b}}^2\right)} \tag{35}$$

provided $\left|\hat{\mathbf{b}}\right| < 1$ and $\hat{a} \neq \beta^{-1}$ where $(\hat{a},\hat{b},\hat{c})$ are possibly disequilibrium average weights. As before, the serial correlation is given by $\hat{b}$. The process of prices satisfies

$$p_t = \hat{\mathbf{b}}p_{t+1} + \frac{1+\beta\hat{\mathbf{c}}}{1-\beta\hat{\mathbf{a}}}u_t \tag{36}$$

so that $p_t$ is exactly as before a public signal of the future price. Nevertheless, the precision of the signal $(\hat{b}\ (1+\beta\hat{\mathbf{c}})\ \ (1-\beta\hat{\mathbf{a}}))^{-2}$now depends also on the weight $\hat{c}$ put on the additional private signal.

The information on the future price that is available to agents is composed now by a public signal and two private signals, namely the current price, the individual perceptions of the $\infty$-PF forecast and the current disturbance. The strategy to compute the equilibrium requires an additional step. In fact, to deal with the solution of the informational choice, we need to define the equilibrium variances of the signals received. Since, the additional private signal $u_t + \eta_{i,t|t}$ only refines the public information conveyed by the current price, the best information embodied by the joint weighting of the two pieces of information $p_t$ and $u_t + \eta_{i,t|t}$ has to be equivalent to the informativeness of a new partially correlated signal $\chi_i$ centered on $p_{t+1}$. This is to say that the presence of a signal on the current disturbance simply increases the precision of the public information conveyed through the current price. Hence, one has to determine the precision of the private information as the variance of the new private signal $\chi_i$. That is quite straightforward since again all the signals are normally distributed. Given the solution to this problem, one can finally recover the restrictions on the profile of all individual weights $\{\hat{a}_i,\hat{b}_i,\hat{c}_i\}_I$ that calibrate agents' forecasts imposing orthogonality conditions on the forecast errors. These are the steps to the proof of the following proposition.

**Proposition 7** *At the limit $T \to \infty$, a unique REE stationary price process (34) and expectations paths (33) exists characterized by*

$$\hat{a}_i = \hat{\mathbf{a}} = \frac{1+\sigma_{(t)}}{\sigma_{(t)}}\frac{\hat{\mathbf{b}}\left(1-\beta^{-1}\hat{\mathbf{b}}\right)}{1-\hat{\mathbf{b}}^2} \quad \textit{and} \quad \hat{c}_i = \hat{\mathbf{c}} = \frac{\hat{\mathbf{b}}\left(\hat{\mathbf{b}}-\beta\right)}{\sigma_{(t)}\beta\left(1-\hat{\mathbf{b}}\beta\right)} \tag{37}$$

*where*

$$\hat{b}_i = \hat{\mathbf{b}} = \frac{1+\kappa-\sqrt{(1+\kappa)^2-4\kappa\beta^2}}{2\kappa\beta}, \tag{38}$$

*with the informational choice $\left(\sigma,\sigma_{(t)}\right)$ satisfying*

$$\sigma = \frac{\kappa\left(1-\hat{\mathbf{b}}^2\right)\sigma_p\sigma_{(t)}}{\sigma_{(t)}-\kappa\hat{\mathbf{b}}^2} \tag{39}$$

*and $\sigma_{(t)} > \kappa\hat{b}^2$.*

**Proof.** Since, the signal $u_t + \eta_{i,t|t}$ only refines the information conveyed by the current price, the joint contribution of these two pieces of information is equivalent to the informativeness of a signal $\chi_i$ centered on $p_{t+1}$. Such a signal can be written as

$$\chi_i \equiv \hat{\mathbf{b}}^{-1} p_t - \frac{\hat{\mathbf{b}}^{-1}}{1+\sigma_{(t)}} \frac{1-\beta\hat{\mathbf{c}}}{1-\beta\hat{\mathbf{a}}} \left(u_t + \eta_{i,t|t}\right),$$

whose precision is given by

$$\mathbf{E}\left(\chi_i - p_{t+1}\right)^{-2} = \frac{\left(1+\sigma_{(t)}\right)\hat{\mathbf{b}}^2}{(1-\hat{\mathbf{b}}^2)\sigma_p \sigma_{(t)}}, \tag{40}$$

obtained after substituting for (35).

This is to say that the presence of a signal on the current disturbance simply increases the precision of the public information conveyed through the current price. Therefore, the posterior conditional volatility is now easily expressed as the inverse of the sum of the precision of the prior on the price process $\sigma_p^{-1}$, the precision of the equivalent signal defined just above at (40) and the precision of the signal on the $\infty$-PF inflation forecast $\sigma^{-1}$. So, we have

$$\mathrm{V}\left(p_{t+1}^{\infty}|\omega_{i,t}, p_t\right) = \frac{\sigma\sigma_p\sigma_{(t)}(1-\hat{\mathbf{b}}^2)}{\left(\sigma+\sigma_p\right)\sigma_{(t)} + \hat{\mathbf{b}}^2\left(\sigma - \sigma_p\sigma_{(t)}\right)} \tag{41}$$

where, as expected, $\lim_{\sigma_{(t)}\to\infty} \mathrm{V}\left(p_{t+1}^{\infty}|\omega_{i,t}, p_t\right)$ gives again (26). The a-priori conditional volatility instead is equal to $(1-\hat{b}^2)\sigma_p$ being the inverse of the sum of the prior on the price process $\sigma_p^{-1}$ and the precision of the current price defined as $\hat{b}^2/(1-\hat{b}^2)\sigma_p$. Working out the entropy constraint as before we obtain a relation between the variances of the two signals object of choice given by (39) with the only constraint $\sigma_{(t)} > \kappa\hat{b}^2$ where $\lim_{\sigma_{(t)}\to\infty}\sigma = \kappa\left(1-\hat{\mathbf{b}}^2\right)\sigma_p$ and $\kappa \equiv (e^{2K}-1)^{-1}$. Now it is possible to solve orthogonality restrictions in order to restrict $(\hat{a},\hat{b},\hat{c})$. These are written as

$$p_t \;:\; \hat{\mathbf{b}}\sigma_p - \hat{\mathbf{a}}\sigma_p - \left(\beta^{-1}-\hat{\mathbf{a}}\right)\hat{\mathbf{b}}\sigma_p - \hat{\mathbf{c}}\left(1+\beta\hat{\mathbf{c}}\right)\left(1-\beta\hat{\mathbf{a}}\right)^{-1} = 0, \tag{42}$$

$$p_{t+1}+\eta_{i,t} \;:\; \sigma_p - \mathbf{ab}\sigma_p - \left(\beta^{-1}-\hat{a}_i\right)\hat{\mathbf{b}}\left(\sigma_p+\sigma\right) = 0, \tag{43}$$

and

$$u_t + \eta_{i,t|t} : 0 = \hat{\mathbf{a}}\left(1+\beta\hat{\mathbf{c}}\right)\left(1-\beta\hat{\mathbf{a}}\right)^{-1} + \hat{\mathbf{c}}\left(1+\sigma_\phi\right), \tag{44}$$

for each agent $i$ at each time $t$ with symmetry relations already imposed. After some manipulation we get $\hat{\mathbf{a}}$

$$\hat{\mathbf{a}} = \frac{1+\sigma_{(t)}}{\sigma_{(t)}} \frac{\hat{\mathbf{b}}\left(1-\beta^{-1}\hat{\mathbf{b}}\right)}{1-\hat{\mathbf{b}}^2} \quad \text{and} \quad \hat{\mathbf{c}} = \frac{\hat{\mathbf{b}}\left(\hat{\mathbf{b}}-\beta\right)}{\beta\left(1-\hat{\mathbf{b}}\beta\right)\sigma_{(t)}} \tag{45}$$

as functions of $\hat{b}$ and $\sigma_{(t)}$. Substituting (39) and (45) into (43) we get a fix point equation for $\hat{b}$ that is the same of (31) at the limit of $T \to \infty$. ■

Notice that the price equilibrium path (34) is exactly equal to (10), the one in section 1. In fact, one can easily see at the equilibrium values

$$\frac{1+\beta\hat{\mathbf{c}}}{1-\beta\hat{\mathbf{a}}} = \frac{1}{1-\beta\hat{\mathbf{a}}} = \frac{\hat{\mathbf{b}}^2-1}{\hat{\mathbf{b}}\beta-1} \tag{46}$$

that is, the aggregate (and individual) response to the monetary shock are the same in both cases. This is not surprising since the entropy constraint fixes how much information is possible to acquire on $p_{t+1}^{\infty}$ independently from the number of sources of information. The capacity-constraint only imposes that in equilibrium if $\sigma_{(t)}$ increases $\sigma$ must decrease according to a certain rate. Hence, the exact allocation of information capacity between the $\infty$-PF inflation forecast and the current money supply shock is restricted but not determined as long as agents are just interested in forecasting $p_{t+1}$ no matter the source of their forecast mistake.

## B.2. Announcing the whole series of monetary shocks

Now I will look at the case in which the CB announces all the single future monetary disturbances. In particular I will assume agents when young receive a private signal for each shock belonging to $\hat{\mathbf{u}}_t^{\infty}$ and then make their choice[13]. Notice that in this case the CB has no role in processing the information, so one can think equally about agents having direct access to the information on the current and future shocks.

At time $t$ agent $i$ receives a series of private signals, one for each $u_{t+\tau}$ with $\tau > 0$. Hence agents forecast the future price according to the following linear rule

$$E_t^i p_{t+1} = \bar{a}_i p_t + \left(\bar{c}_i + \beta^{-1}\right) \sum_{\tau=1}^{\infty} \bar{b}_{\tau,i} \left(u_{t+\tau} + \eta_{i,t|t+\tau}\right) + c_i \left(u_t + \eta_{i,t|t}\right) \tag{47}$$

where $(\bar{a}_i, \bar{c}_i)$ play the same role as before whereas $\bar{b}_{\tau,i}$ is the weight given by agent $i$ on his equivalent signal about the innovation at $\tau$ lags from the current time. The strategy to show the equivalence with the previous settings is to restrict the class of the forecasting rule imposing a recursive structure on the weights $\bar{b}_{\tau,i} = \bar{b}_i \bar{b}_{\tau+1,i}$ for each $\tau > 1$ with $\bar{b}_{1,i} = \bar{b}_i$ and looking for an equilibrium of this type. At this point we can rewrite the individual forecasting strategy

[13] In other words they die with their private information. The extension of this setup to the case of infinite living agents should address the issue of how to model memory of rational inattentive agents. There are very few recent attempt to provide a solution.

as

$$E_t^i p_{t+1} = \bar{a}_i p_t + \bar{b}_i \left( \left(c_i + \beta^{-1}\right) \sum_{\tau=0}^{\infty} \bar{b}_i^{\tau} u_{t+1+\tau} + \left(\beta^{-1} - a_i\right) \eta_{i,t} \right) + \bar{c}_i \left(u_t + \eta_{i,t|t}\right) \tag{48}$$

where

$$\bar{\eta}_{i,t} \equiv \frac{(1+\beta\bar{c}_i)}{\bar{b}_i (1-\beta\bar{a}_i)} \sum_{\tau=1}^{\infty} \bar{b}_i^{\tau} \eta_{i,t+\tau|t},$$

is defined as an aggregate shock. Notice that for $(\bar{a}_i, \bar{b}_i, \bar{c}_i) = (\hat{a}_i, \hat{b}_i, \hat{c}_i)$ the forecasting strategy exactly delivers (33) at the equilibrium (use (34)). Nevertheless, notice that imposing a recursive structure to the weights $\bar{b}_{\tau,i}$ equals to impose a structural relation on the variances of the signals $\{\sigma_{(t)}\}_t^{\infty}$ that have to determined in equilibrium. Therefore to conclude the equivalence one has to check that there exist a feasible informational choice that is consistent with the weight restriction and solve the agents' problem at the equilibrium values. The following proposition states a positive answer.

**Proposition 8** *At the limit $T \to \infty$, a REE stationary price process (34) and expectations paths (47) exist characterized by $\bar{a}_i = \hat{\mathbf{a}}$, and $\bar{c}_i = \hat{\mathbf{c}}$ given by (37) and $\bar{b}_i = \hat{\mathbf{b}}$ according to (38) where*

$$\bar{b}_{\tau,i} = \bar{b}_i \bar{b}_{\tau+1,i} \text{ for each } \tau > 1 \text{ and } \bar{b}_{1,i} = \bar{b}_i, \tag{49}$$

*with the informational choice $\{\sigma_{(t)}\}_t^{\infty}$ satisfying*

$$\sigma_{(t+\tau)} = \frac{\kappa\left(1-\mathbf{b}^2\right)\sigma_{(t)}}{\sigma_{(t)} - \kappa\mathbf{b}^2} \quad \forall \tau > 1, \tag{50}$$

*and $\sigma_{(t)} > \kappa\mathbf{b}^2$.*

**Proof.** To prove the proposition we have to show that given the recursive restriction to weights, then the informational choice satisfies the capacity constraints coherently with the orthogonality conditions. We already know that the informational choice has a solution in correspondence of $\bar{\sigma} = \sigma$ with $\sigma_{t|t} > \kappa\bar{\mathbf{b}}^2$according to (39) where $\bar{\sigma}$ is the variance of $\bar{\eta}_{i,t}$. The orthogonality conditions instead are written as

$$p_t \quad : \quad \bar{\mathbf{b}}\sigma_p - \bar{\mathbf{a}}\sigma_p - \left(\beta^{-1} - \bar{\mathbf{a}}\right)\bar{\mathbf{b}}\sigma_p - \bar{c}_i\left(1+\beta\bar{\mathbf{c}}\right)\left(1-\beta\bar{\mathbf{a}}\right)^{-1} = 0 \tag{51a}$$

$$u_{t+\tau} + \eta_{i,t+\tau|t} \quad : \quad \bar{\mathbf{b}}^{\tau-1} - \bar{\mathbf{a}}\bar{\mathbf{b}}^{\tau} - \left(\beta^{-1} - \bar{\mathbf{a}}\right)\bar{\mathbf{b}}^{\tau}\left(1+\sigma_{(t+\tau)}\right) = 0, \forall\, \tau > 1, \tag{51b}$$

$$u_t + \eta_{i,t|t} \quad : \quad \bar{\mathbf{a}}\left(1+\beta\bar{\mathbf{c}}\right)\left(1-\beta\bar{\mathbf{a}}\right)^{-1} + \bar{\mathbf{c}}\left(1+\sigma_{(t)}\right) = 0. \tag{51c}$$

where individual weights equal to average weights. From the previous case we know that $(\bar{\mathbf{a}}, \bar{\mathbf{b}}, \bar{\mathbf{c}}) = (\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}})$ satisfies the three conditions above. Notice that the recursive restrictions requires that $\sigma_{(t+\tau)} = \sigma_{(t+1+\tau)}$ for each $\tau > 1$. In such a case, in equilibrium it is

$$\bar{\sigma} = \frac{(1+\beta\bar{\mathbf{c}})^2}{\bar{\mathbf{b}}_i^2\,(1-\beta\bar{\mathbf{a}})^2}\sum_{\tau=1}^{\infty}\bar{\mathbf{b}}_i^{2\tau}\sigma_{(t+\tau)}$$

or simply $\sigma_{(t+\tau)} = \bar{\sigma}/\sigma_p$, so that the precision on the information about future shocks is given by $\sigma_{(t+\tau)} = \kappa(1-\bar{\mathbf{b}}^2)\,\sigma_{t|t}(\sigma_{t|t}-\kappa\bar{\mathbf{b}}^2)$ with $\sigma_{t|t} > \kappa\bar{\mathbf{b}}^2$ in analogy with the previous case. ■

It is important to remark on the generality of the recursive restriction that I used to solve the problem. This is one way among infinite others to restrict the indeterminacy in the information choice problem that also showed up in the previous case. In particular, the recursive restriction to the weights implies that agents learn about each future disturbance with the same precision $\sigma_{(t+\tau|t)}^{-1}$. Then, they use the rest of their current information capacity to learn only about the current realization. It is of course possible to find different patterns of information acquisition, but the forecasting performance in terms of posterior conditional volatility would not change as it is fixed from the outset by $\kappa$.

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