

---

DOCUMENT  
DE TRAVAIL  
N° 413

---

**OPTIMAL MONETARY AND PRUDENTIAL POLICIES**

Fabrice Collard, Harris Dellas, Behzad Diba and Olivier Loisel

December 2012



**OPTIMAL MONETARY AND PRUDENTIAL POLICIES**

Fabrice Collard, Harris Dellas, Behzad Diba and Olivier Loisel

December 2012

Les Documents de travail reflètent les idées personnelles de leurs auteurs et n'expriment pas nécessairement la position de la Banque de France. Ce document est disponible sur le site internet de la Banque de France « [www.banque-france.fr](http://www.banque-france.fr) ».

Working Papers reflect the opinions of the authors and do not necessarily express the views of the Banque de France. This document is available on the Banque de France Website “[www.banque-france.fr](http://www.banque-france.fr)”.

# Optimal Monetary and Prudential Policies\*

Fabrice Collard<sup>†</sup>    Harris Dellas<sup>‡</sup>    Behzad Diba<sup>§</sup>    Olivier Loisel<sup>¶</sup>

---

\*Version of December 3, 2012. We are grateful to Pierpaolo Benigno, Nobu Kiyotaki, Franck Smets, Javier Suarez, Skander Van den Heuvel and Mike Woodford, as well as to our discussants Ivan Jaccard, Enrico Perotti and Cédric Tille, for valuable suggestions. The views expressed in this paper are those of the authors and should not be interpreted as reflecting those of the Banque de France.

<sup>†</sup>Department of Economics, University of Bern. [fabrice.collard@gmail.com](mailto:fabrice.collard@gmail.com), <http://fabcol.free.fr>

<sup>‡</sup>Department of Economics, University of Bern, CEPR. [harris.dellas@vwi.unibe.ch](mailto:harris.dellas@vwi.unibe.ch), <http://www.harrisdellas.net>

<sup>§</sup>Department of Economics, Georgetown University. [dibab@georgetown.edu](mailto:dibab@georgetown.edu), <http://www9.georgetown.edu/faculty/dibab/>

<sup>¶</sup>CREST (ENSAE). [olivier.loisel@ensae.fr](mailto:olivier.loisel@ensae.fr), <http://www.cepremap.ens.fr/depot/pageperso/LOIOLI/>

**Abstract:** The recent financial crisis has highlighted the interconnectedness between macroeconomic and financial stability and has raised the question of whether and how to combine the corresponding main policy instruments (interest rate and bank-capital requirements). This paper offers a characterization of the jointly optimal setting of monetary and prudential policies and discusses its implications for the business cycle. The source of financial fragility is the socially excessive risk-taking by banks due to limited liability and deposit insurance. We characterize the conditions under which locally optimal (Ramsey) policy dedicates the prudential instrument to preventing inefficient risk-taking by banks; and the monetary instrument to dealing with the business cycle, with the two instruments co-varying negatively. Our analysis thus identifies circumstances that can validate the prevailing view among central bankers that standard interest-rate policy cannot serve as the first line of defense against financial instability. In addition, we also provide conditions under which the two instruments might optimally co-move positively and countercyclically.

**Key Words:** Prudential policy, Capital requirements, Monetary policy, Ramsey-optimal policies.

**JEL Classification:** E32, E44, E52.

**Résumé :** la récente crise financière, en faisant apparatre les liens entre stabilité macroéconomique et stabilité financière, a soulevé la question de la façon dont combiner les principaux instruments de politique correspondants (taux d'intérêt et contraintes de capital bancaire). Ce papier propose une caractérisation des politiques monétaire et prudentielle conjointement optimales et en discute les implications pour le cycle d'affaires. L'origine de la fragilité financière est la prise de risque excessive des banques due à leur responsabilité limitée et à l'assurance des dépôts. Nous caractérisons les conditions sous lesquelles la politique localement optimale (au sens de Ramsey) assigne à l'instrument prudentiel la tâche d'empêcher une prise de risque inefficace par les banques ; et à l'instrument monétaire la tâche de gérer le cycle d'affaires, les deux instruments co-variant négativement. Notre analyse identifie ainsi des circonstances qui peuvent justifier le point de vue prédominant parmi les banquiers centraux selon lequel une politique de taux d'intérêt standard ne peut pas servir de première ligne de défense contre l'instabilité financière. De plus, nous donnons également des conditions sous lesquelles les deux instruments pourraient, à l'optimum, co-varier positivement et de manière contracyclique.

**Mots-clés :** politique prudentielle, contraintes de capital, politique monétaire, politiques optimales au sens de Ramsey.

**Codes JEL :** E32, E44, E52.

# 1 Introduction

Monetary and prudential policies have traditionally been designed and analyzed in isolation from one another. The 2007-2009 financial crisis, however, has aroused interest in the study of their interactions. Policymakers [e.g., Bernanke (2010), Blanchard et al. (2010)] have commented on the extent to which monetary policy can or should address concerns about financial stability. An important aspect of prudential policy is the forthcoming introduction of state-contingent bank capital requirements [Basel Committee on Banking Supervision (2010)]. Policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012)] have summarized alternative views about the potential substitutability or complementarities between state-contingent bank capital requirements and the interest rate set by monetary policy. There is a general presumption that both policies will be counter-cyclical most of the time, but policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)] have also envisioned scenarios that may put the two policies at odds with each other over the business cycle.

In this paper, we develop a New Keynesian model with banks to study the jointly optimal settings of monetary policy and a prudential policy that sets state-dependent bank-capital requirements. While there exists no complete treatment of jointly optimal monetary and prudential policies in the literature, recent work is making progress on this front. Existing work predominantly emphasizes the credit cycle, the “risk-taking channel” of monetary policy [as discussed, for example, in Borio and Zhu (2008)], and pecuniary externalities. It typically views excessive risk taking in terms of the aggregate volume of credit. Angeloni and Faia (2011), for example, consider a link between the bank leverage ratio and the risk of bank runs; Christensen, Meh and Moran (2011) postulate an externality that links the riskiness of bank projects to the ratio of aggregate credit to GDP.<sup>1</sup> In these models, economic expansions – following, for example, a favorable productivity shock or a period of low interest rates – lead to excessive risk taking and call for a policy response that may be either monetary or prudential.

The view that excessive risk taking is the *outcome* of credit expansion and that monetary policy can be used to avert financial crises seems to be widely held.<sup>2</sup> Nonetheless, the role of credit expansion in *causing* the financial crisis is not firmly established empirically, while it is uncontroversial that financial stress is associated with a deterioration in the quality of the portfolios of financial institutions. This is why the present paper adopts an alternative framework that relates risk-taking behavior to well known institutional features of the economy (namely, limited liability and deposit insurance) but does not require that excessive risk taking be manifested also in the volume of loans made by banks in addition to the quality of banks’ portfolios.<sup>3</sup>

More specifically, we introduce aggregate risk into a variant of Van den Heuvel’s (2008) model of

---

<sup>1</sup>The link between excessive risk taking and the volume of credit is also present in a number of contributions that abstract from monetary policy [e.g., Bianchi (2011), Bianchi and Mendoza (2010), Jeanne and Korinek (2010)] or from prudential policy [Benigno et al. (2011)]. In these contributions, a pecuniary externality associated with a collateral constraint plays a central role: it makes an asset-price boom increase the value of borrowers’ collateral, which in turn feeds the boom. Kashyap, Berner and Goodhart (2011) emphasize the relevance of the downside of pecuniary externalities (contractions accompanied by fire sales of assets) for the design of prudential policies.

<sup>2</sup>For instance, Schularick and Taylor (2012, p. 1032) claim that banking crises are “credit booms gone wrong.”

<sup>3</sup>Martinez-Miera and Suarez (2012) examine capital requirements from a perspective similar to ours, but abstract from monetary policy. Gete and Tiernan (2011) consider the role of capital requirements in Hachem’s (2010) model of over-lending (as we elaborate below), also abstracting from monetary policy.

optimal capital requirements, and we embed the resulting model in a DSGE framework with sticky prices and monetary policy. As in Van den Heuvel (2008), who follows on this point a branch of the micro-banking literature [surveyed, for instance, by Freixas and Rochet (2008)], the need for capital requirements arises from limited liability and deposit insurance. These institutional features truncate the distribution of risky returns facing investors, the banks lending to these investors, and the depositors funding the banks; this is the externality that leads to excessive risk taking. This perspective, unlike the contributions that emphasize the credit cycle, does not automatically link excessive risk taking to the *volume* of credit. In our model, excessive risk taking involves the *type* of projects that banks may be tempted to fund because limited liability protects them from incurring large losses. Therefore, a productivity boom does not lead to excessive risk taking as long as it does not make the funding of risky projects relatively more lucrative than the funding of safe projects.

Sufficiently high capital requirements can always force banks to internalize the riskiness of their loans and thus tame risk-taking behavior. But monetary policy may not be suited to this task as it works primarily through the volume rather than the composition of credit. In our benchmark model, due to the assumption of perfectly competitive banks operating under constant returns to scale, the interest rate has no effect on risk-taking incentives as it affects the cost of funding *all* (safe or risky) projects equally. From this vantage point, capital requirements and the interest rate are sharply distinct policy tools that do not affect the same margins: monetary policy affects the volume but not the type of credit, while prudential policy affects both the type and the volume of credit. This makes monetary policy ineffective in ensuring financial stability. So, in contrast to models that emphasize the credit cycle, our framework does not suggest a strong connection between interest-rate policy and financial stability. As such, our framework accords with the standard view among policymakers [expressed, for instance, in Bernanke (2010)] that standard interest-rate policy cannot serve as the first line of defense against financial instability.

We determine (numerically) a monetary policy and (analytically) a prudential policy that are jointly locally Ramsey-optimal. In so doing, we improve on the existing literature [e.g., Angeloni and Faia (2011), Christensen, Meh, and Moran (2011)], which (numerically) determines some monetary and prudential policies that are jointly optimal in a more restrictive sense.<sup>4</sup>

Our optimal policy sets the capital requirement to the minimum level that prevents inefficient risk taking by banks. We show that this policy is locally Ramsey-optimal in the following sense. Setting the capital requirement *just below* this threshold level is not optimal because it triggers a discontinuous increase in the amount of inefficient risk taken by banks. This discontinuity is due to our deposit-insurance and limited-liability assumptions, which make banks' expected excess return convex in the amount of risk that they take. And setting the capital requirement *just above* this threshold level is not optimal because it has a negative first-order effect on welfare that cannot be offset by any change in the interest rate around its optimal value (as this change would have a zero first-order effect on welfare). This negative first-order effect on welfare, in turn, is due to the fact that taxes on banks' profits distort banks' funding decisions as they make equity finance more expensive than

---

<sup>4</sup>Indeed, in this literature, the deviations of the interest rate and the capital requirement from their steady-state values are optimized within some parametric families of simple rules, and the steady-state value of the capital requirement is not locally optimal.

debt finance for the banks. This tax distortion implies that raising the capital requirement above the threshold level decreases the (bank-loan-financed) capital stock, which is already inefficiently low due to monopolistic competition and the tax distortion itself.<sup>5</sup>

This optimal capital requirement is state dependent: it rises in response to shocks that increase banks' incentives to fund risky projects. In our benchmark model, the interest rate and the capital requirement do not affect the same margins, so there is a clear-cut optimal division of tasks between monetary and prudential policies: In response to shocks that do not affect banks' risk-taking incentives, prudential policy should leave the capital requirement constant, and monetary policy should move the interest rate in a standard way. In response to shocks that increase (decrease) banks' risk-taking incentives, prudential policy should raise (cut) the capital requirement, and monetary policy should cut (raise) the interest rate in order to mitigate the effects of prudential policy on bank lending and output. In the latter case, optimal prudential policy is pro-cyclical (as it is the proximate cause of the contraction of output), while optimal monetary policy is counter-cyclical. So, with this chain of causality, the two policies move in opposite directions over the cycle – a situation envisaged by some policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)].

The *qualitative* properties of jointly optimal policies, however, can change when we modify our benchmark model to allow monetary policy to affect the risk-taking incentives of banks.<sup>6</sup> To illustrate this, we develop an extension in which the cost of originating and monitoring safe loans is an increasing function of the aggregate volume of such loans.<sup>7</sup> Consequently, all the shocks that affect the volume of safe loans also affect the cost of such loans and thus banks' risk-taking incentives. A favorable productivity shock, for instance, raises the volume and hence the cost of safe loans, and this higher cost increases banks' risk-taking incentives. Following this shock, optimal prudential policy raises the capital requirement, and optimal monetary policy raises the interest rate; but the optimal interest-rate hike is smaller than it would be in our benchmark model because optimal monetary policy mitigates the effects of the rise in the capital requirement on bank lending and output. In this case, the jointly optimal monetary and prudential policies are thus both counter-cyclical.<sup>8</sup>

The rest of the paper is organized as follows. Section 2 presents our benchmark model. Section 3 discusses our analytical results on prudential policy, with proofs relegated to the Appendix. Sections 4 and 5 discuss our calibration and report our numerical results for the locally Ramsey-optimal monetary and prudential policies in the benchmark model. Section 6 presents two extensions (one with an externality in the cost of banking, the other with correlated shocks) that seem relevant for policy concerns. Section 7 contains concluding remarks.

---

<sup>5</sup>An alternative to our model with the tax distortion would be to follow Van den Heuvel (2008) and model the cost of raising capital requirements as foregone liquidity from holding bank deposits. In his model, liquid deposits and equity are the only sources of funding for bank loans. So, when capital requirements are higher, banks don't issue as much liquid deposits, and households suffer a loss of utility. We don't pursue this track because commercial paper (rather than liquid deposits) is a more likely marginal source of funding for US banks, as Cúrdia and Woodford (2009) point out. For the same reason, following Cúrdia and Woodford (2009) and others, our modelling of optimal monetary policy will abstract from the transactions frictions that motivate the Friedman Rule.

<sup>6</sup>The standard claim in the literature is that a prolonged period of low interest rates may lead banks to make more risky loans, for instance because they run out of less risky lending opportunities.

<sup>7</sup>We use this ad-hoc assumption about costs of banking to keep the extension brief. Hachem (2010) develops a full model of this type of externality in bank lending.

<sup>8</sup>There are also other ways to make both policies optimally countercyclical in our setup. As an example, we will present a case with correlated shocks.

## 2 Benchmark Model

To motivate the role of banks in our model, we assume that households must sell their unfurnished capital stock to capital producers –who need to borrow the necessary funds– at the end of each period and buy back the furnished capital at the beginning of the next period. The capital producers have access to two alternative technologies to furnish capital: one is safe and the other risky. The latter technology is less efficient on average, but limited liability tempts the capital producers to use it. Banks are needed to monitor the producers who claim to use the safe technology, to ensure that they do so. Banks themselves, however, may have adverse incentives due to limited liability and deposit insurance, and these adverse incentives give a role to prudential policy.

Each period is divided into two subperiods. At the beginning of the first subperiod, all exogenous shocks are realized, except one, and these realizations are observed by all agents. The only shock that is not realized at the beginning of the first subperiod is the binary shock leading to the success or failure of the risky technology (in the case of failure, forcing any capital producers using this technology to default on their bank loans). This shock is realized at the end of the second subperiod, after households, firms, and banks have made their optimal decisions.

### 2.1 Households

Preferences are defined by the discount factor  $\beta \in (0, 1)$  and the period utility

$$U(c_t, h_t) = \log(c_t) - \frac{1}{1+\chi} h_t^{1+\chi}$$

over consumption  $c_t$  and hours of work  $h_t$ , where  $\chi > 0$ . Households maximize  $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$ . All household decisions are taken in the first subperiod of each period  $t$ . We assume that, during this subperiod, households own the furnished capital stock  $k_t$  and rent it, at the rental price  $z_t$ , to intermediate goods producers. At the end of the subperiod, after production has taken place, households get back  $(1 - \delta)k_t$  worn-out capital from intermediate goods producers, where  $0 < \delta < 1$ , and invest  $i_t$  in new capital. Unfurnished capital  $x_t$ , made of both worn-out capital and new capital, has to be furnished before it can be used for production next period. So, at this stage, households sell their unfurnished capital

$$x_t = (1 - \delta)k_t + i_t, \tag{1}$$

at the price  $q_t^x$ , to capital goods producers, who can furnish it in the second subperiod of period  $t$ . At the beginning of the next period, households buy furnished capital  $k_{t+1}$ , at a price  $q_{t+1}$ , from capital goods producers.

Households also acquire  $s_t$  shares in banks at a price  $q_t^b$ . These banks are perfectly competitive and last for only one period.<sup>9</sup> Households face the budget constraint

$$c_t + d_t + q_t^b s_t + q_t k_t + i_t = w_t h_t + \frac{1 + R_{t-1}^D}{\Pi_t} d_{t-1} + s_{t-1} \omega_t^b + z_t k_t + q_t^x x_t + (\omega_t^k + \omega_t^f - \tau_t^h), \tag{2}$$

---

<sup>9</sup>Our assumption that banks last for only one period implies that there is no durable relationship between lenders and borrowers, which simplifies the financial contract.



where  $d_t$  represents the real value of bank deposits with a gross nominal return  $R_t^D$ ,  $\Pi_t = \frac{P_t}{P_{t-1}}$  is the gross inflation rate in the price index for consumption,  $w_t$  is the real wage,  $\omega_t^k$  and  $\omega_t^f$  represent the profits of capital producers and firms producing intermediate goods,  $\omega_t^b$  stands for dividends paid by banks, and  $\tau_t^h$  is a lump-sum tax paid by households.<sup>10</sup>

Households choose  $(c_t, h_t, d_t, s_t, k_t, i_t, x_t)_{t \geq 0}$  to maximize utility subject to (1) and (2). The first-order conditions for optimality are:

$$\begin{aligned}
\frac{1}{c_t} &= \lambda_t, \\
\lambda_t &= \beta (1 + R_t^D) E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}, \\
h_t^x &= \lambda_t w_t, \\
\lambda_t q_t^x &= \lambda_t^k, \\
\lambda_t &= \lambda_t^k, \\
\lambda_t (q_t - z_t) &= \lambda_t^k (1 - \delta), \\
\lambda_t q_t^b &= \beta E_t \{ \lambda_{t+1} \omega_{t+1}^b \},
\end{aligned} \tag{3}$$

where  $E_t \{ \cdot \}$  denotes the expectation operator conditional on the information available in the first subperiod of period  $t$ , which includes the realization of all the aggregate shocks except the binary shock leading to the success or failure of the risky technology. The optimality conditions imply in particular

$$\begin{aligned}
q_t^x &= 1, \\
q_t &= 1 - \delta + z_t.
\end{aligned}$$

## 2.2 Intermediate goods producers

There is a unit mass of monopolistically competitive firms producing intermediate goods. Firm  $j$  operates the production function:

$$y_t(j) = h_t(j)^{1-\nu} k_t(j)^\nu \exp(\eta_t^f),$$

where  $0 < \nu < 1$ ,  $k_t(j)$  is capital rented by firm  $j$ , and  $\eta_t^f$  is an exogenous productivity shock. We assume that firms set their prices facing a Calvo-type price rigidity (with no indexation). Since their optimization problem is standard, we don't present the details. We let  $\alpha$  denote the probability that a firm does not get to set a new price at a given date.

The firms' cost minimization problem implies

$$\frac{z_t}{w_t} = \left( \frac{\nu}{1-\nu} \right) \left[ \frac{h_t(j)}{k_t(j)} \right].$$

<sup>10</sup>We do not need to model equity stakes in firms as we assume that the representative household owns these firms forever.

### 2.3 Final goods producers

Producers of the final good are perfectly competitive and aggregate the intermediate goods  $y_t(j)$  to form the final good  $y_t$ . The production function is given by

$$y_t = \left( \int_0^1 y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad (4)$$

where  $\sigma > 1$ . Profit maximization leads to the demand for good  $j$

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} y_t, \quad (5)$$

and free entry lead to the price index

$$P_t = \left( \int_0^1 P_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (6)$$

The final good may be used for consumption, investment, the monitoring of firms, and government purchases.

### 2.4 Capital goods producers

The capital producing firms are owned by households and are perfectly competitive. They buy unfurnished capital  $x_t$  during the second subperiod of period  $t$  to produce furnished capital  $k_{t+1}$  that they sell to households at the price  $q_{t+1}$  in the first subperiod of period  $t+1$ . Each capital producer chooses to operate either a safe technology (S) or a risky technology (R). Those choosing technology S use  $x_t^S$  units of unfurnished capital to produce  $k_{t+1}^S$  units of furnished capital with

$$k_{t+1}^S = x_t^S. \quad (7)$$

Producers choosing technology R are subject to an aggregate shock  $\theta_t$  that is independent of all the other shocks. When  $\theta_t = 0$ , they produce nothing. More specifically, they use  $x_t^R$  units of unfurnished capital to produce

$$k_{t+1}^R = \theta_t \exp(\eta_t^R) x_t^R$$

units of furnished capital, with

$$\begin{aligned} \theta_t &= 0 \text{ with probability } \phi_t, \\ \theta_t &= 1 \text{ with probability } 1 - \phi_t, \end{aligned}$$

where  $\phi_t$  is the exogenous stochastic probability of failure and  $\eta_t^R$  is the exogenous stochastic productivity if the project is successful. We assume that the realization of  $\eta_t^R$  is always positive ( $\eta_t^R > 0$ ), so that in the absence of failure, the risky technology is more productive than the safe one. Producers choose whether to use technology S or technology R after observing the realization of  $\eta_t^R$  and  $\phi_t$  (which occur at the beginning of the first subperiod), but before observing the realization of  $\theta_t$  (which occurs at the end of the second subperiod).

We assume that using the risky technology is always inefficient<sup>11</sup>, but capital producers have limited liability and may have an incentive to hide the fact that they use the risky technology. There is therefore a need to monitor capital producers who claim to use the safe technology. We further assume that only banks have the appropriate monitoring skills. This motivates a setup with capital producers getting funds from banks to buy unfurnished capital.

More specifically, the risky technology is inefficient in the sense that, for all realizations of  $\phi_t$ ,  $\eta_t^R$  and  $\Psi_t$ ,

$$(1 - \phi_t) \exp(\eta_t^R) \leq 1 - \Psi_t, \quad (8)$$

where  $\Psi_t > 0$  is the exogenous marginal resource cost of monitoring a capital producer who claims to use the safe technology.<sup>12</sup> The left-hand side of (8) represents the marginal benefit of allocating one unit of unfurnished capital to the risky technology (the expected output of this technology). The right-hand side is the opportunity cost, which is the output of the safe technology net of the monitoring cost.

We consider loan contracts between capital goods producers and banks. Our motivation for considering loan contracts is that in reality non-bank firms prefer debt finance because they get a tax deduction. They also need some equity, presumably because of the agency problem associated with debt. In our model, for simplicity, we abstract from this agency problem and capital goods producers have no equity. So this translates into a framework in which their funding is entirely with loans and they pay no tax. Thus, a capital producer  $i$  choosing technology  $j \in \{S, R\}$  borrows

$$q_t^x x_t^j(i) = l_t^j(i) \quad (9)$$

at a nominal interest rate  $R_t^j$ .<sup>13</sup> Since capital producers have limited liability, those using the risky technology will default on their loans in the event of failure (when  $\theta_t = 0$ ).

A producer  $i$  using technology S chooses  $x_t^S(i)$  to maximize

$$\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ q_{t+1} k_{t+1}^S(i) - \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(i) \right] \right\}$$

subject to (7) and (9). The optimality condition implies

$$E_t \{ \lambda_{t+1} q_{t+1} \} = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} (1 + R_t^S) q_t^x. \quad (10)$$

A producer  $i$  using technology R chooses  $x_t^R(i)$  to maximize

$$(1 - \phi_t) \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ q_{t+1} \exp(\eta_t^R) x_t^R(i) - \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(i) \right] \middle| \theta_t = 1 \right\}$$

subject to (9), where  $E_t \{ \cdot | \theta_t = 1 \}$  denotes the expectation operator conditional on the information available in the first subperiod of period  $t$  and on the success of the risky technology in the second

<sup>11</sup>Our focus in this paper, as in the extant literature, is not on the amount of risk per se but rather on the amount of socially excessive risk. Abstracting from socially desirable risk greatly simplifies the analysis as it removes the need to also solve in general equilibrium for the optimal amount of risk.

<sup>12</sup>In Section 6, we will consider an extension of the model in which  $\Psi_t$  is endogenous.

<sup>13</sup>There is no need to work with nominal loan contracts in our model. However, since we will assume that monetary policy sets a nominal interest rate, and for the sake of realism, we make loan contracts nominal.

subperiod of period  $t$ . The optimality condition implies

$$E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \} \exp(\eta_t^R) = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} (1 + R_t^R) q_t^x. \quad (11)$$

Since our model allows for two distinct interest rates, banks need to monitor the capital producers that borrow at the lower rate to ensure that they use the associated technology. Our model has no equilibrium with  $R_t^R < R_t^S$ .<sup>14</sup> Therefore, there is no need for banks to monitor capital producers that claim to use the risky technology. Accordingly, we will associate a cost with monitoring capital producers that claim to use the safe technology.

As usual, with constant returns to scale, the first-order conditions imply that firms make zero profits. When both (10) and (11) hold, capital producers are indifferent between the two technologies and

$$\frac{1 + R_t^R}{1 + R_t^S} = \frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\}} \exp(\eta_t^R). \quad (12)$$

If the interest-rate ratio on the left-hand side is strictly higher than the critical value on the right-hand side, then capital producers use only technology S.

## 2.5 Banks

Banks are owned by households. They are perfectly competitive. They incur a cost  $\Psi_t l_t^S$  of monitoring safe loans, where  $\Psi_t$  satisfies (8). They can fund their loans by raising equity ( $e_t$ ) or issuing deposits ( $d_t$ ). They make safe and risky loans ( $l_t^S$  and  $l_t^R$ ). Their balance-sheet identity is

$$l_t^S + l_t^R = e_t + d_t, \quad (13)$$

as  $e_t$  is defined net of monitoring costs.

We assume that banks can hide risky loans in their portfolio from regulators up to an exogenous fraction  $\gamma_t$  of their safe loans.<sup>15</sup> Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between  $\theta_t$  and other shocks), risky projects reduce welfare. Therefore, the prudential authority optimally forbids banks to choose  $l_t^R > \gamma_t l_t^S$ , so that banks face the following constraint:

$$l_t^R \leq \gamma_t l_t^S. \quad (14)$$

The prudential authority also imposes a capital requirement in the form of a minimum ratio of equity to loans:

$$e_t \geq \kappa_t (l_t^S + l_t^R). \quad (15)$$

In the first subperiod of period  $t + 1$ , regulators close the banks that cannot meet their deposit obligations: the banks with

$$\frac{1 + R_t^S}{\Pi_{t+1}} l_t^S + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R - \frac{1 + R_t^D}{\Pi_{t+1}} d_t < 0,$$

<sup>14</sup>Indeed, if we had  $R_t^R < R_t^S$ , then funding the safe projects would strictly dominate funding the risky projects because it would pay more in every state (whatever the realization of  $\theta_t$ ) and incur no monitoring cost.

<sup>15</sup>Our model can be extended to allow regulators to choose  $\gamma_t$  by incurring some supervision cost, as in Van den Heuvel (2008), but we do not pursue this dimension of optimal policy.

or equivalently, using (13), those with

$$e_t < - \left( \frac{R_t^S - R_t^D}{1 + R_t^D} \right) l_t^S - \left( \theta_t \frac{1 + R_t^R}{1 + R_t^D} - 1 \right) l_t^R.$$

When  $l_t^R = 0$  or  $\theta_t = 1$ , the right-hand side of this inequality is negative as long as lending rates are above the deposit rate, which will be the case in equilibrium because loans either incur a monitoring cost or entail a risk for banks. When  $l_t^R > 0$  and  $\theta_t = 0$ , the right-hand side of this inequality is positive if and only if

$$l_t^R > e_t + \left( \frac{R_t^S - R_t^D}{1 + R_t^D} \right) l_t^S.$$

We want our model to capture the fact that banks find equity finance more costly than debt finance in reality. We attribute this to a tax distortion (tax deduction for debt finance), although this interpretation is not essential for our analysis. We take this distortionary tax to be a feature of the environment: the model does not explain why this tax is in place, and the policymakers in our model (the monetary and prudential authorities) cannot set this tax optimally.<sup>16</sup>

The particular way we specify the tax distortion (and the timing of the tax deduction for monitoring costs) ensures that unanticipated changes in the price level cannot cause insolvency.<sup>17</sup> The banks in our model may be insolvent only if they extend too many risky loans, and the risky projects fail. Specifically, we assume that gross revenues from loans are taxed at the constant rate  $\tau$  after deductions for gross payments on deposits and monitoring costs. The amount of bank equity, net of monitoring costs, is therefore  $e_t = q_t^b s_t - (1 - \tau) \Psi_t l_t^S$ .

The representative bank chooses  $e_t$ ,  $d_t$ ,  $l_t^R$  and  $l_t^S$  to maximize

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1 - \tau) \omega_{t+1}^b}{\lambda_t} \right\} - e_t - (1 - \tau) \Psi_t l_t^S,$$

where

$$\omega_{t+1}^b = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R - \frac{1 + R_t^D}{\Pi_{t+1}} d_t \right\}, \quad (16)$$

subject to (13), (14) and (15).

Note that, for any non-negative value of  $\kappa_t$  (therefore  $e_t$ ), we have

$$\omega_{t+1}^b = \theta_t \left( \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S + \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R - \frac{1 + R_t^D}{\Pi_{t+1}} d_t \right) + (1 - \theta_t) \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S - \frac{1 + R_t^D}{\Pi_{t+1}} d_t \right\}.$$

## 2.6 Government and market-clearing conditions

The government has exogenous purchases  $G_t$  and guarantees bank deposits. The lump-sum tax on households balances the budget.<sup>18</sup>

<sup>16</sup>This feature of the tax code seems to be one of the primary reasons for banks to lobby against higher capital requirements, at least in the US and the euro area. It is commonly invoked in models with both debt and equity finance [e.g. Jermann and Quadrini (2009, 2012)], to break the Modigliani-Miller theorem about irrelevance of financial structure. We motivate our modeling choice in the conclusion.

<sup>17</sup>In our setting with one-period competitive banks incurring real monitoring costs and extending nominal loans, a change in the price level could lead to insolvency. We don't think this is an interesting feature of the model and have specified our "tax code" to rule it out.

<sup>18</sup>It is harmless to abstract from deposit insurance fees paid by banks and include these in the lump-sum tax paid by households who own the banks.

The losses imposed by bank  $j$  on the deposit insurance fund amount to

$$\zeta_t(j) = \max \left\{ 0, \frac{1 + R_{t-1}^D}{\Pi_t} d_{t-1}(j) - \frac{1 + R_{t-1}^S}{\Pi_t} l_{t-1}^S(j) - \delta_{t-1} \frac{1 + R_{t-1}^R}{\Pi_t} l_{t-1}^R(j) \right\},$$

and the lump-sum tax paid by households is

$$\tau_t^h = G_t + \int_0^1 \{ \zeta_t(j) - \tau[\omega_t^b(j) + \Psi_t l_t^S(j)] \} dj.$$

We consider two policy instruments: the deposit rate  $R_t^D$  for monetary policy and the capital requirement  $\kappa_t$  for prudential policy. We will discuss our specifications of prudential policy in Sections 3 and 5. For each specification, our monetary policy will be the Ramsey-optimal policy.

Firms producing intermediate goods rent their capital from the representative household; in equilibrium, their choices must satisfy

$$\int_0^1 k_t(j) dj = k_t.$$

Similarly obvious market-clearing conditions must be satisfied in the markets for labor, loans, and unfurnished capital. The market-clearing condition for goods is

$$c_t + i_t + G_t + \Psi_t l_t^S = y_t.$$

### 3 Prudential Policy

This section derives conditions for prudential policy to rule out equilibria with risk taking and ensure the existence of equilibria without risk taking. We first show that our model can only have equilibria at the two corners with  $l_t^R = 0$  and  $l_t^R = \gamma_t l_t^S$ , and that the capital constraint is binding in any equilibrium. Next, we consider a benchmark prudential policy that internalizes the externality (arising from limited liability) by making banks the residual claimants to any losses they may incur. We then characterize the least stringent prudential policy that rules out risk taking, and show that it is locally Ramsey-optimal.

#### 3.1 Ruling out candidate equilibria

We focus on symmetric equilibria in which all banks have the same loan portfolio. We will also assume throughout that the following condition holds:

$$\frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \leq 1. \quad (17)$$

This condition seems plausible because failure of risky projects at date  $t$  leads to destruction of the capital stock at date  $t + 1$ , and this by itself should increase both the price of capital ( $q_{t+1}$ ) and the marginal utility of consumption ( $\lambda_{t+1}$ ). However, this condition amounts to an implicit restriction on the set of policies that we consider, as it presumes that policies will not overturn the qualitative effects of the failure of risky projects.

We first show that the banks' optimization problem rules out the existence of equilibria with  $0 < l_t^R < \gamma_t l_t^S$ . The basic insight follows Van den Heuvel (2008), but since we have added aggregate risk and made other changes to his model, we prove the following proposition in the Appendix.

**Proposition 1:** *There are no equilibria with  $0 < l_t^R < \gamma_t l_t^S$ . When  $0 < l_t^R < \gamma_t l_t^S$ , (a) if banks go bankrupt ( $\omega_{t+1}^b = 0$ ) when risky projects fail ( $\theta_t = 0$ ), then banks can increase their market value by tilting the loan portfolio towards more risky loans; (b) if banks do not go bankrupt ( $\omega_{t+1}^b > 0$ ) when risky projects fail ( $\theta_t = 0$ ), then they can increase their market value by tilting the loan portfolio towards more safe loans.*

The intuition follows. If, given the loan portfolio, bank equity is sufficiently small to be wiped out when risky projects fail, then banks do not internalize the cost of additional risk taking. Additional losses from increasing  $l_t^R$ , if risky projects fail, are truncated by deposit insurance and limited liability. Consequently, the only candidate for an equilibrium with the possibility of bank failure involves the corner solution  $l_t^R = \gamma_t l_t^S$ .

Alternatively, if bank equity is sufficiently large for banks to remain solvent even when risky projects fail, then banks internalize the cost of additional risk taking. In that case, since we assume that the risky technology is inefficient, banks can increase their market value by reducing  $l_t^R$ . Accordingly, the only candidate for an equilibrium without the possibility of bank failure involves the corner solution  $l_t^R = 0$ . In particular, if bank equity is large enough to make banks residual claimants on their risky loans when  $l_t^R = \gamma_t l_t^S$ , then there does not exist an equilibrium with  $l_t^R = \gamma_t l_t^S$ .

Next, we show that there are no equilibria in which the capital constraint is lax:

**Proposition 2:** *In equilibrium, the capital constraint is binding:*

$$e_t = \kappa_t (l_t^S + l_t^R). \quad (18)$$

This Proposition follows almost directly from our assumption about the tax advantage of debt finance over equity finance, but we provide a proof in the Appendix.

### 3.2 A benchmark policy

Proposition 1 leads to a sufficient condition for prudential policy to rule out equilibria with  $l_t^R > 0$  and ensure the existence of an equilibrium with  $l_t^R = 0$ : the capital requirement can be sufficiently high to make any bank the residual claimant to the potential losses arising from funding risky projects. This benchmark policy is characterized by the following proposition:

**Proposition 3:** *(a) A sufficient condition for existence and uniqueness of an equilibrium and for  $l_t^R = 0$  in this equilibrium is that*

$$\kappa_t > \tilde{\kappa}(R_t^D, R_t^S) \equiv 1 - \frac{1}{1 + \gamma_t} \frac{1 + R_t^S}{1 + R_t^D}; \quad (19)$$

(b) in this equilibrium,

$$\tilde{\kappa}(R_t^D, R_t^S) = \tilde{\kappa}_t \equiv \frac{(1-\tau)(\gamma_t - \Psi_t)}{\tau + (1-\tau)(1+\gamma_t)}; \quad (20)$$

(c)  $\tilde{\kappa}_t$  is increasing in  $\gamma_t$ , and decreasing in  $\Psi_t$ .

We prove this proposition in the Appendix, by considering a given bank  $j$  that takes the maximum amount of risk ( $l_t^R(j) = \gamma_t l_t^S(j)$ ). We show that this bank will remain solvent when risky projects fail ( $\theta_t = 0$ ) if and only if (19) holds. We then use the banks' optimality conditions at the equilibrium with  $l_t^R = 0$  to express  $\tilde{\kappa}(R_t^D, R_t^S)$  in terms of parameters and exogenous shocks and obtain (20).

We assume  $\gamma_t > \Psi_t$ , which implies  $\tilde{\kappa}_t > 0$ , so that condition (20) may or may not be met depending on the value of  $\kappa_t$ . This restriction states that the temptation to take risk would be present if banks were not subject to any (positive) capital requirements. The threshold  $\tilde{\kappa}_t$  is increasing in  $\gamma_t$ : the higher the fraction of risky loans that a deviating bank can hide, the riskier this bank, and the higher the capital requirement needed to make it remain solvent in case of failure. And  $\tilde{\kappa}_t$  is decreasing in  $\Psi_t$ : the higher the cost of monitoring safe loans, the higher the spread between the interest rate on safe loans and that on deposits; thus, the larger the cash flow from safe loans that is available to redeem the deposits, and the lower the capital requirement needed to make a deviating bank remain solvent in case of failure.

Although this benchmark policy suffices to ensure the existence of an equilibrium without risk taking, we show next that it is more stringent than necessary and that the least stringent policy ensuring the existence of an equilibrium without risk taking is locally Ramsey-optimal in our model.

### 3.3 The locally optimal policy

We now derive a necessary and sufficient condition for prudential policy to ensure the existence of an equilibrium with  $l_t^R = 0$ , and then show that the least stringent policy satisfying this condition is locally Ramsey-optimal.

Consider a bank  $j$  that deviates from a candidate equilibrium with  $l_t^R = 0$  to take the maximum amount of risk ( $l_t^R(j) = \gamma_t l_t^S(j)$ ). There exists an equilibrium with  $l_t^R = 0$  if and only if this deviating bank has a negative expected excess return. In the Appendix, we derive the threshold value of  $\kappa_t$  that makes its expected excess return negative, and we prove the following proposition:

**Proposition 4:** (a) A necessary and sufficient condition for existence of an equilibrium with  $l_t^R = 0$  is  $\kappa_t \geq \kappa_t^*$ , where

$$\kappa_t^* \equiv (1-\tau) \frac{(1-\phi_t)\gamma_t [\exp(\eta_t^R) - 1] + \Psi_t [(1-\phi_t)\gamma_t \exp(\eta_t^R) - \phi_t]}{\phi_t(1+\gamma_t) - \gamma_t\tau(1-\phi_t) [\exp(\eta_t^R) - 1]}; \quad (21)$$

(b)  $\kappa_t^* < \tilde{\kappa}_t$ ; (c)  $\kappa_t^*$  is decreasing in the probability of failure of the risky technology  $\phi_t$ , and increasing in the productivity of the risky technology conditionally on its success  $\eta_t^R$ .

The derivations in the Appendix consider a bank  $j$  that contemplates a deviation from a candidate equilibrium with  $l_t^R = 0$ . The same intuition we gave for Proposition 1 (roughly) applies: if there are



profitable deviations, the most profitable one is at the corner with maximum risk ( $l_t^R(j) = \gamma_t l_t^S(j)$ ). To derive the value of  $\kappa_t^*$ , we make bank  $j$  indifferent between staying at the safe corner and moving to the risky corner. The bank turns indifferent with less equity at stake than what would make it residual claimant (i.e., we have  $\kappa_t^* < \tilde{\kappa}_t$ ) because the bank has incurred monitoring costs and has a vested interest in remaining solvent to recoup these costs. In a way, monitoring costs in our model work like giving the banks some charter value that they would like to preserve by avoiding bankruptcy.

The preceding intuition also helps us understand the nature of the state dependence, in our model, of the constraint  $\kappa_t \geq \kappa_t^*$ . Macro-prudential policy must be tight enough to prevent risk taking in equilibrium. The threshold  $\kappa_t^*$  depends negatively on the probability of failure of the risky technology  $\phi_t$  because failure risk, by itself, makes risk-taking less attractive. Similarly,  $\kappa_t^*$  rises with the productivity of the risky technology conditionally on its success  $\eta_t^R$  because a higher  $\eta_t^R$  increases the temptation to finance risky, rather than safe, projects.

Perhaps a more surprising feature of (21) is that  $\kappa_t^*$  does not depend on the monetary policy instrument  $R_t^D$ . This is because, in our model, the deposit rate  $R_t^D$  does not affect banks' incentives for risk taking. In particular, it does not affect the spread between the interest rate on risky loans  $R_t^R$  and the interest rate on safe loans  $R_t^S$ . In a way, this is not a surprising feature for a model with perfect competition and constant returns. Our banks never run out of safe projects to fund and always end up making zero profits. This is the opposite extreme from arguments that (explicitly or implicitly) postulate a fixed number of potential projects and thereby link more lending with more risk taking (as banks run out of safe lending opportunities). We will revisit this contrast between extreme modelling assumptions in Section 6.

Turning to normative implications, let  $(R_\tau^{D*})_{\tau \geq 0}$  denote the monetary policy that is Ramsey-optimal when the prudential policy is  $(\kappa_\tau^*)_{\tau \geq 0}$ . The following proposition states that, under a certain condition, setting jointly  $(R_\tau^D)_{\tau \geq 0}$  to  $(R_\tau^{D*})_{\tau \geq 0}$  and  $(\kappa_\tau)_{\tau \geq 0}$  to  $(\kappa_\tau^*)_{\tau \geq 0}$  is locally Ramsey-optimal:

**Proposition 5:** *If the right derivative of welfare with respect to  $\kappa_t$  at  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  is strictly negative for all  $t \geq 0$ , then the policy  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  is locally Ramsey-optimal.*

We prove this proposition in the Appendix. The intuition is the following. First, whatever  $R_t^D$  in the neighborhood of  $R_t^{D*}$ , setting  $\kappa_t$  just below  $\kappa_t^*$  is not optimal, because it triggers a discontinuous increase in the amount of risk taken by banks. Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between  $\theta_t$  and other shocks), this discontinuous increase in the amount of risk has a discontinuous negative effect on welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that  $(R_t^D, \kappa_t)$  is close enough to  $(R_t^{D*}, \kappa_t^*)$ . Second, if the right derivative of welfare with respect to  $\kappa_t$  at  $(R_t^{D*}, \kappa_t^*)$  is strictly negative, then setting  $\kappa_t$  just above  $\kappa_t^*$  is not optimal either, because it has a negative first-order effect on welfare that cannot be offset by any change in  $R_t^D$  around its optimal value  $R_t^{D*}$  (as this change would have a zero first-order effect on welfare).

The right derivative of welfare with respect to  $\kappa_t$  at  $(R_t^{D*}, \kappa_t^*)$  can be expected to be strictly negative because increasing  $\kappa_t$  from  $\kappa_t^*$  decreases the capital stock, which is already inefficiently low due to the

monopoly and tax distortions, without reducing the amount of risk, which is already zero. We check numerically, for the calibration considered in the next section, that this derivative is indeed strictly negative. This derivative is equal to the Lagrange multiplier associated to the constraint  $\kappa_t = \kappa_t^*$  in the optimization problem that determines  $R_t^{D*}$ . We first use the program Get Ramsey developed by Levin and López-Salido (2004) and used in Levin, Onatski, Williams and Williams (2005) to get analytically the non-linear first-order conditions of this optimization problem. We then use Dynare to solve numerically, at the first order, the resulting system of constraints and first-order conditions, and thus get the first-order approximation of this Lagrange multiplier (among other variables). We check that this Lagrange multiplier is strictly negative at the steady state, which implies that it is strictly negative for small enough shocks. We also check that it is strictly negative at the first order in the presence of shocks of a standard size.

### 3.4 Ruling out equilibria with $l_t^R = \gamma_t l_t^S$

We next formulate a prudential feedback rule that precludes equilibria with  $l_t^R = \gamma_t l_t^S$ , and coincides with  $\kappa_t = \kappa_t^*$  in equilibrium. That is, under this rule, there is a unique equilibrium and, in this equilibrium,  $l_t^R = 0$  and  $\kappa_t$  takes the minimum value that is consistent with  $l_t^R = 0$ .

We will assume throughout that the following condition holds:

$$\frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \mid \theta_t = 1 \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}} \leq 1. \quad (22)$$

This condition seems plausible but, as we noted in our discussion of (17), it amounts to an implicit restriction on the set of policies that we consider.<sup>19</sup>

We prove the following proposition in the Appendix:

**Proposition 6:** *Under the prudential-policy rule*

$$\kappa_t = \frac{1 - \phi_t}{\phi_t} \frac{\gamma_t}{1 + \gamma_t} \frac{R_t^R - R_t^S}{1 + R_t^D} + \frac{1}{\phi_t} \frac{\gamma_t}{1 + \gamma_t} \Psi_t - \frac{R_t^S - R_t^D}{1 + R_t^D}, \quad (23)$$

*there exists a unique equilibrium and, in this equilibrium,  $l_t^R = 0$  and  $\kappa_t = \kappa_t^*$ .*

Although the formal proof in the Appendix takes a different approach, a heuristic rendition is to start with the equilibrium at the safe corner and define  $R_t^R$  as the highest rate that a deviating bank could charge on a loan to a risky firm. In this case, (23) just states  $\kappa_t^*$  as a function of interest-rate spreads. It gives the critical value of  $\kappa_t$  for making the bank indifferent between staying at the safe corner (where all the other banks are) and jumping to the risky corner. The critical value is fairly intuitive. The first two terms represent the temptation to deviate from the safe corner to the risky corner: a deviating bank will pocket  $R_t^R - R_t^S$  if risky projects succeed (with probability  $1 - \phi_t$ ) and

<sup>19</sup>The condition seems plausible when we consider the pricing of a bond with default risk—a bond that pays \$1 when risky projects succeed and pays nothing when they fail. The inequality (22) says this risky bond has a higher expected real return, compared to a nominal bond with no default risk, in the equilibria we consider.

save monitoring costs. The third term represents the opportunity cost  $R_t^S - R_t^D$  of this deviation when risky projects fail (with probability  $\phi_t$ ).

So, this feedback rule suffices for keeping banks at the safe corner. In the Appendix we show that it also suffices to rule out an equilibrium at the risky corner, because the safe corner becomes even more attractive to an individual bank if there is a mass of banks at the risky corner (in which case the risk is priced).

## 4 Calibration

The parameters pertaining to households and firms are standard. The period of time is a quarter. The discount rate is such that the household discounts the future at the deposit rate, 2.76% per year. The labor supply elasticity is set to 1. Markups are set to 10%, which implies a value of the elasticity of substitution between intermediate goods of 11. The capital elasticity in the intermediate-good technology is set such that the labor share is 0.66, implying a value for  $\nu$  of 0.34. The depreciation rate,  $\delta$ , is set to 0.025 which corresponds to a 10% annual depreciation rate. Firms are assumed to reset their prices every 4 quarters on average, implying the value 0.75 for the Calvo parameter  $\alpha$ .

Table 1: Calibration

Parameter	Description	Value
Preferences		
$\beta$	Discount factor	0.993
$\chi$	Inverse of labor supply elasticity	1.000
Technology		
$\nu$	Capital elasticity	0.340
$\sigma$	Elasticity of substitution	11.00
$\delta$	Depreciation rate	0.025
Nominal rigidities		
$\alpha$	Price stickiness	0.750
Banking (steady state)		
$\tau$	Tax rate	0.023
$\kappa^*$	Capital requirement	0.080
$\Psi$	Marginal monitoring cost	0.006
$\phi$	Failure probability	0.031
$\gamma$	Maximal risky/safe loans ratio	0.356
$\eta^R$	Productivity of the risky technology	1.005
Shock processes		
$\rho$	Persistence	0.950

The parameters pertaining to the banking system are set as follows. Using variables without a time subscript to denote steady-state values,  $\eta^R$  is set such that the annualized lending rate on risky projects is 2% higher than that on safe projects in the steady state. We assume that the steady-state yield differential  $R^S - R^D$  is 3.16% per annum. The tax rate on bank profits is set to 0.023. This value is chosen to equate the after-tax return on bank equity in our model to the after-tax return in US

data.<sup>20</sup> Our calibration of the optimal steady-state capital requirement,  $\kappa^*$ , is 0.08. The steady-state monitoring cost,  $\Psi$ , is set such that the first-order condition of the representative bank is satisfied:

$$\Psi = \frac{R^S - R^D}{1 + R^D} - \frac{\tau\kappa}{1 - \tau}.$$

This yields the value  $\Psi = 0.006$ . The steady-state failure probability of risky projects is set such that the model matches the average failure rate in the US economy (0.86% per quarter). This leads to a steady-state value for  $\phi_t$  of 0.031. The maximal risky/safe loans ratio is then obtained by solving the optimal capital requirement equation,  $\kappa = \kappa^*$ , yielding

$$\gamma = \frac{\phi(1 - \tau)\Psi + \kappa}{(1 - \phi)(1 - \tau(1 - \kappa)) + (1 - \tau)\Psi(R^R - R^S) - \phi\kappa} = 0.356.$$

The persistence of all the shocks is set to  $\rho = 0.95$ .<sup>21</sup> For the impulse-response functions presented in the next section, we set the innovations to the technology shock  $\eta_t^f$  and the fiscal shock  $G_t$  equal to 1%, and the innovation to  $\Psi_t$  to 10%. We set the innovation to  $\eta_t^R$  such that the annualized risk premium increases from 2% to 3%. And we set the innovation to  $\phi_t$  such that the probability of failure increases by 1/3 of a percent.

## 5 Numerical Results

We consider two alternative prudential policies. Our benchmark prudential policy sets  $\kappa_t$  equal to its locally Ramsey-optimal value  $\kappa_t^*$ , which is 0.08 at the steady state. The other policy keeps  $\kappa_t$  constant at 0.10. This value is high enough, given the size of our shocks, to keep the economy in the safe equilibrium.

For each of these prudential policies, we solve for the Ramsey monetary policy using Dynare and the program Get Ramsey developed by Levin and López-Salido (2004). In both cases, the optimal steady-state inflation rate is zero, given the presence of Calvo-type price rigidity and the absence of monetary distortions in our model.

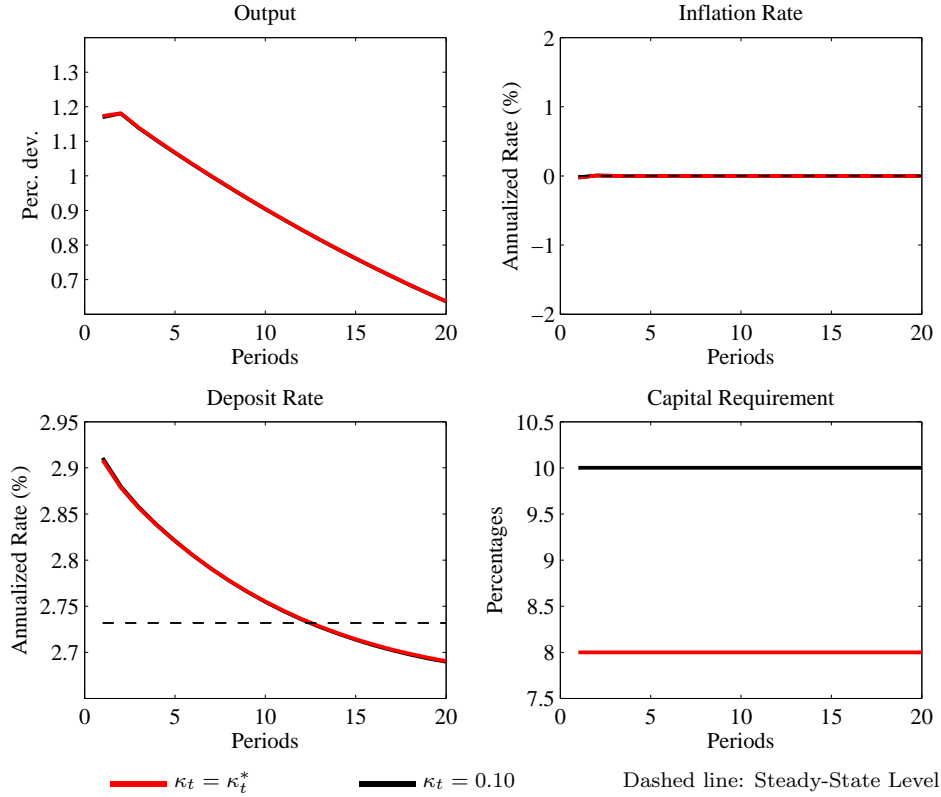
Figure 1 displays the optimal responses to a favorable productivity shock (positive innovation to  $\eta_t^f$ ). The responses, with the exception of those of the interest rates, are expressed as percentage deviations from each steady state. The response of the interest rates is measured in terms of the level of the interest rate as a percentage per annum (rather than a deviation from the steady state). The horizontal dashed line corresponds to the steady-state level of the interest rate, so values below this line represent accommodative monetary policy following the shock, and values above represent restrictive monetary policy.

Since a productivity shock does not create a temptation to take more risk in our model, it does not affect the optimal capital requirement  $\kappa_t^*$ . So the optimal responses of the policy rate, output

<sup>20</sup>In the data, the after-tax return on equity is given by  $(1 - \tau^c)\pi/e$  where  $\tau^c$ ,  $\pi$  and  $e$  respectively denote corporate tax rate, profits and equity. In our model, this quantity is given by  $(1 - \tau)(\pi + e)/e - 1$  where  $\tau$  denotes the proper tax rate that applies in our model. By equating these two quantities, and using the fact that the average return on equity is 7% and the tax rate on corporate profits is 35%, we obtain the number reported in Table 1.

<sup>21</sup>Note that in order to study the response of the economy to shocks to the failure rate, we assume that  $\phi_t = (1 + \exp(-(u_t - v)))^{-1}$  where  $u_t$  is assumed to follow a zero-mean AR(1) process and  $v = 3.3253$ .

Figure 1: Response to a Favorable Productivity Shock ( $\eta_t^f$ )



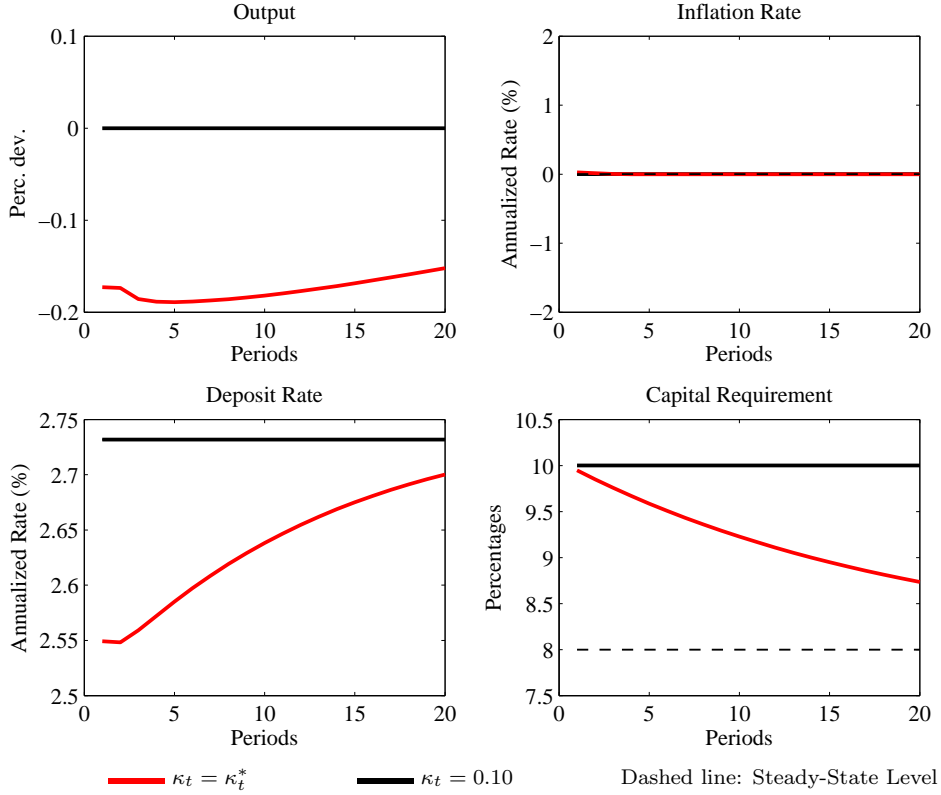
and inflation are the same, regardless of the prudential policy in place ( $\kappa_t = \kappa_t^*$  or  $\kappa_t = 0.10$ ). These optimal responses to a productivity shock are qualitatively similar to optimal responses in the benchmark New-Keynesian (NK) model with capital. Optimal policy essentially keeps inflation at zero. This requires an increase in the deposit rate for a while, because the natural real interest rate rises in the model with capital.<sup>22</sup> Optimal responses to an increase in government purchases (not reported here) are also similar to those from the NK model, and independent of prudential policy.

A positive shock to  $\eta_t^R$  is a pure temptation for banks and firms to deviate from the safe equilibrium; it increases the return on risky projects in case they succeed. Figure 2 shows that this shock increases the capital requirement under the optimal prudential policy ( $\kappa_t = \kappa_t^*$ ). By itself, the tightening of capital requirements increases the cost of banking in our model. The optimal monetary-policy response is to cut the deposit rate in order to curb the increase in bank lending rates. The overall effects on output are small, and inflation is essentially zero under optimal policy.

We find this thought experiment quite useful in the context of policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012), Macklem (2011), Wolf (2012), Yellen (2010)] of how monetary and prudential policies may be substitutes for each other or move to offset each other's effects. In our case, one policy is contractionary and the other expansionary in order to manage risk-taking incentives with the smallest possible adverse effects on investment.

<sup>22</sup>Both the favorable productivity shock and the resulting increase in employment increase the marginal product of capital.

Figure 2: Response to an Increase in the Productivity of the Risky Technology ( $\eta_t^R$ )



The same observations apply to optimal responses to shocks to the probability of failure of the risky technology ( $\phi_t$ ) and the maximal risky/safe loans ratio ( $\gamma_t$ ). These shocks affect the economy only through their effect on the optimal capital requirement  $\kappa_t^*$ , which in turn calls for a monetary-policy response to mitigate the macroeconomic effects. Instead of presenting these responses, which are qualitatively the same as those of Figure 2, we present the effects of an exogenous tightening of the capital requirement. Figure 3 shows the responses to an increase in  $\kappa_t$  by one percent (from 0.080 to 0.088). The optimal monetary-policy response is to cut the annualized deposit rate by about 10 basis points. Again, the overall decrease in output is small (less than 0.1% at the trough) and inflation remains at zero under optimal policy.

Figure 4 shows responses to a change in the marginal cost of making safe loans  $\Psi_t$ . In contrast to the other shocks, this shock has direct macroeconomic effects in addition to its effects on the risk-taking incentives of banks. Under a prudential policy keeping  $\kappa_t$  constant, this shock reduces output in our model, and monetary policy cuts the deposit rate to mitigate this effect. Under the optimal prudential policy ( $\kappa_t = \kappa_t^*$ ), output falls more because, as we explained earlier, the increase in  $\kappa_t$  (needed to prevent risk taking) increases bank lending rates. Monetary policy reacts to the tighter capital requirements by cutting the deposit rate further.

Our model highlights a distinction across policy instruments that we think deserves more emphasis than it gets in the existing literature: changes in the capital requirement can directly manage risk-taking incentives, while changes in the policy interest rate cannot. When the capital requirement

Figure 3: Response to an Exogenous Tightening of the Capital Requirement ( $\kappa_t$ )

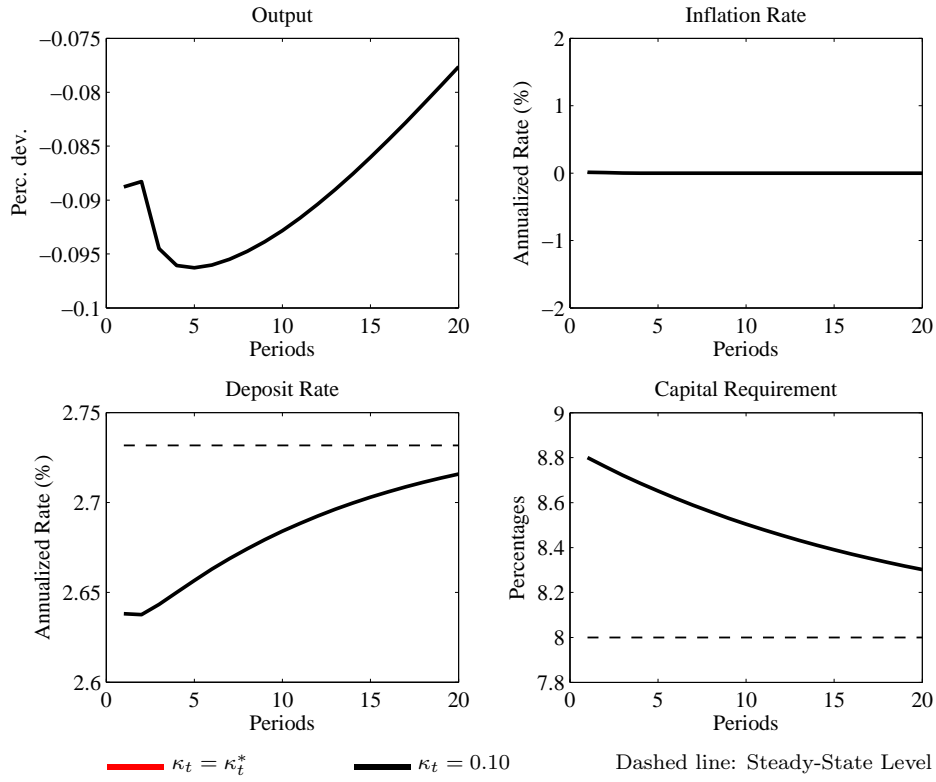
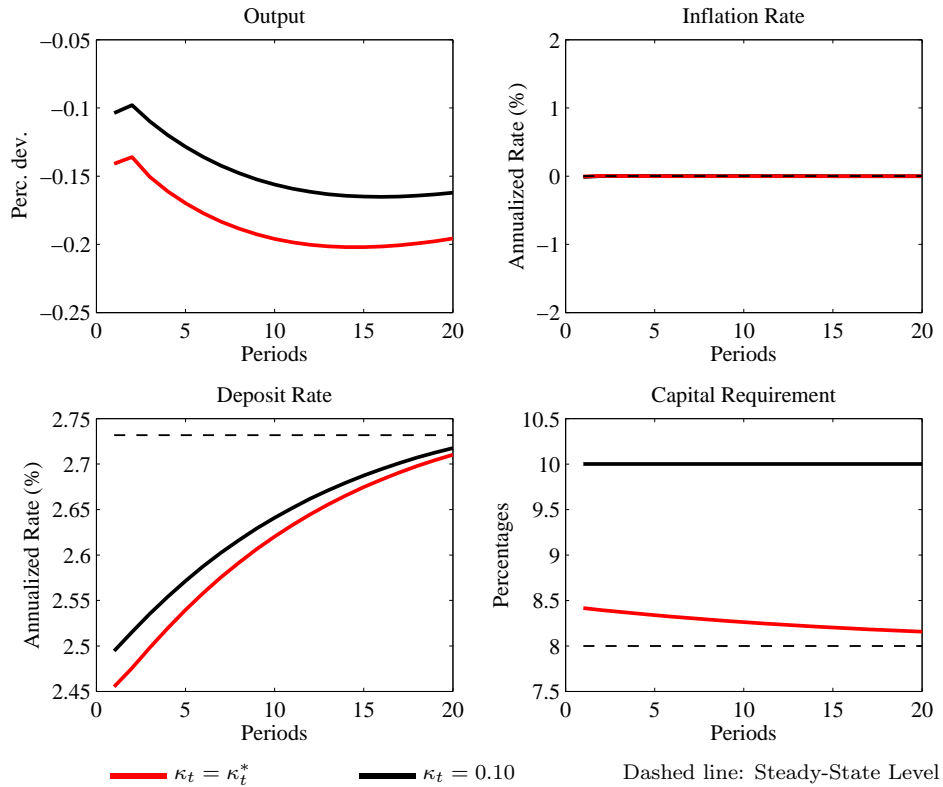


Figure 4: Response to an Increase in the Cost of Making Safe Loans ( $\Psi_t$ )



risers to curb risk taking, a contraction ensues, and the policy interest rate is cut. With this chain of causality, optimal prudential policy is pro-cyclical, and optimal monetary policy is counter-cyclical.

Nonetheless, our model also provides a framework for thinking about some scenarios (or extensions) that can make optimal prudential policy counter-cyclical, as we discuss below.

## 6 Extensions and Policy Concerns

Our benchmark model, while stylized, provide several useful insights. For example, as Angeloni and Faia (2011) elaborate, the leading argument for Basel III-type counter-cyclical capital requirements is the observation that default risk rises during recessions; and risk-weighted (Basel II-type) capital requirements automatically tighten policy in recessions, unless the regulatory rate is lowered.<sup>23</sup> Our model suggests a reason for the latter to happen, that is, for cutting capital requirements when default risk is high: When the banks have enough skin in the game, the additional risk makes banks less inclined to fund risky projects, allowing prudential policy to set lower requirements without undermining the stability of the banking system.

In this subsection, we illustrate how (admittedly ad hoc) extensions can provide additional insights. We consider two extensions: externalities in bank lending, and correlation across shocks affecting the incentives to take risks and shocks to the business cycle.

### 6.1 An externality

Our model assumes perfect competition and constant returns in the banking sector. As we noted earlier these assumptions imply that shocks that directly affect the optimal policy interest rate (like standard productivity or fiscal shocks) do not affect the optimal bank-capital requirement. We now consider a simple (ad-hoc) extension that links the cost of banking to the aggregate volume of safe loans and thus allows such shocks to affect both policy margins. Hachem (2010) develops a model with an externality in banking costs. In her model, banks ignore the effect of their own lending decision on the pool of borrowers, with heterogeneous levels of risk, that is available to other banks. Here, we will only consider a simple example of such an externality— we keep the example simple to preserve our earlier derivations that treated  $\Psi_t$  as exogenous to the banks' decisions. Specifically, we assume

$$\log(\Psi_t) = \log(\Psi) + \varrho [\log(l_t^S) - \log(l^S)] \quad (24)$$

where the term  $\log(l_t^S) - \log(l^S)$  is the log-deviation of the aggregate volume of safe loans from its steady-state value, and  $\varrho = 0$  corresponds to our benchmark model. We show the impulse responses for  $\varrho = 0, 1$ , and 5. Figure 5 illustrates the effects of a favorable productivity shock. Following this shock, optimal prudential policy raises the capital requirement, while optimal monetary policy is less restrictive (raises the deposit rate by less, and later on cuts it by more) than in the benchmark model. The reason why optimal monetary policy is more accommodative during a productivity boom is that

<sup>23</sup>See Covas and Fujita (2010) for a quantitative assessment of the procyclical effects of bank capital requirements under Basel II.



optimal prudential policy turns more contractionary.<sup>24</sup>

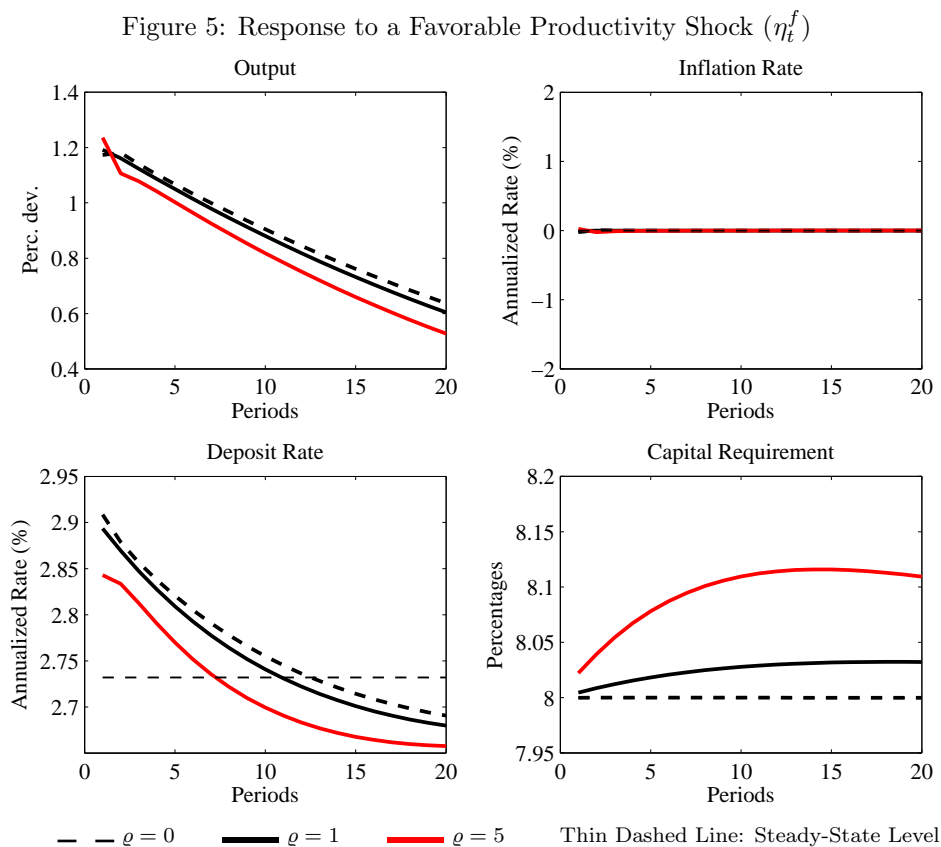


Figure 6 shows the optimal responses to an increase in the risk of failure of the risky technology. Absent the externality (looking at the dashed lines in the figure), optimal prudential policy cuts the capital requirement because banks are naturally less tempted to take risk, while optimal monetary policy raises the deposit rate to curb the expansionary effects of prudential policy. With the externality, the expansion creates a temptation to take more risk (as the cost of making safe loans increases). So, optimal prudential policy cuts the capital requirement by less, and optimal monetary policy raises the deposit rate by less. Figure 7, which is the analogue to Figure 2, makes a similar point about responses to an increase in  $\eta_t^R$ : with the externality, optimal prudential policy increases the capital requirement by less, and optimal monetary policy cuts the deposit rate by less. In terms of optimal output fluctuations in Figures 5–7, the externality always dampens the optimal response (expansion or contraction) of output.

<sup>24</sup>The increase in capital requirements that is necessary to prevent banks from taking risks is larger than the increase that would exactly offset the banks' tendency to lend too much as they ignore the effect of their own lending decision on monitoring costs.

Figure 6: Response to an Increase in the Risk of Failure of the Risky Technology ( $\phi_t$ )

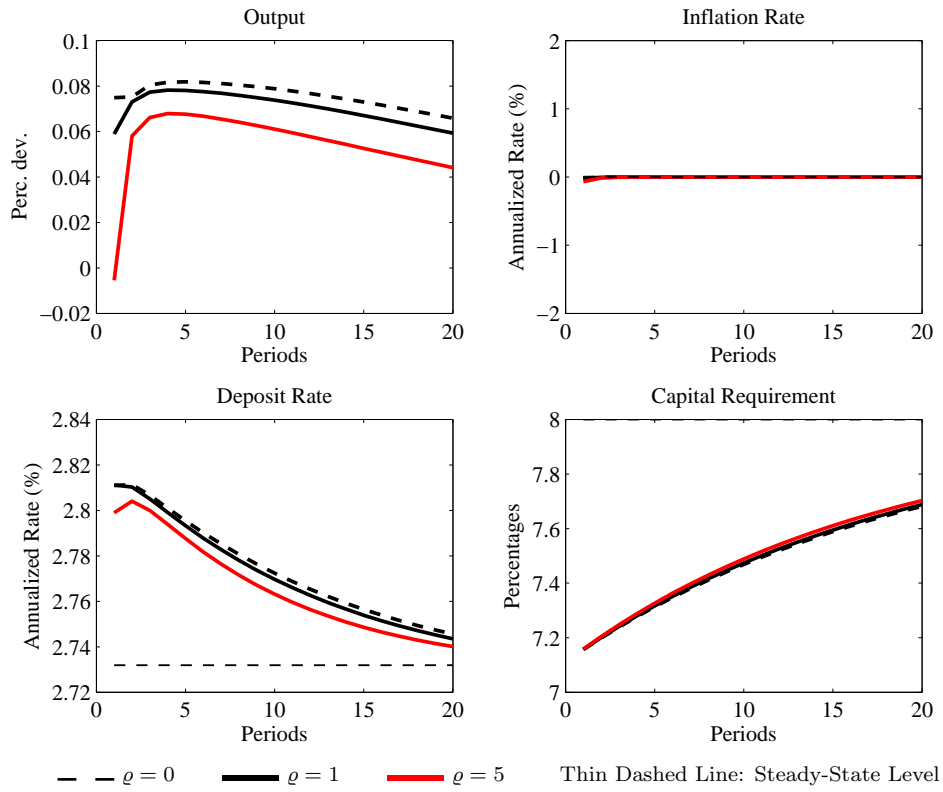
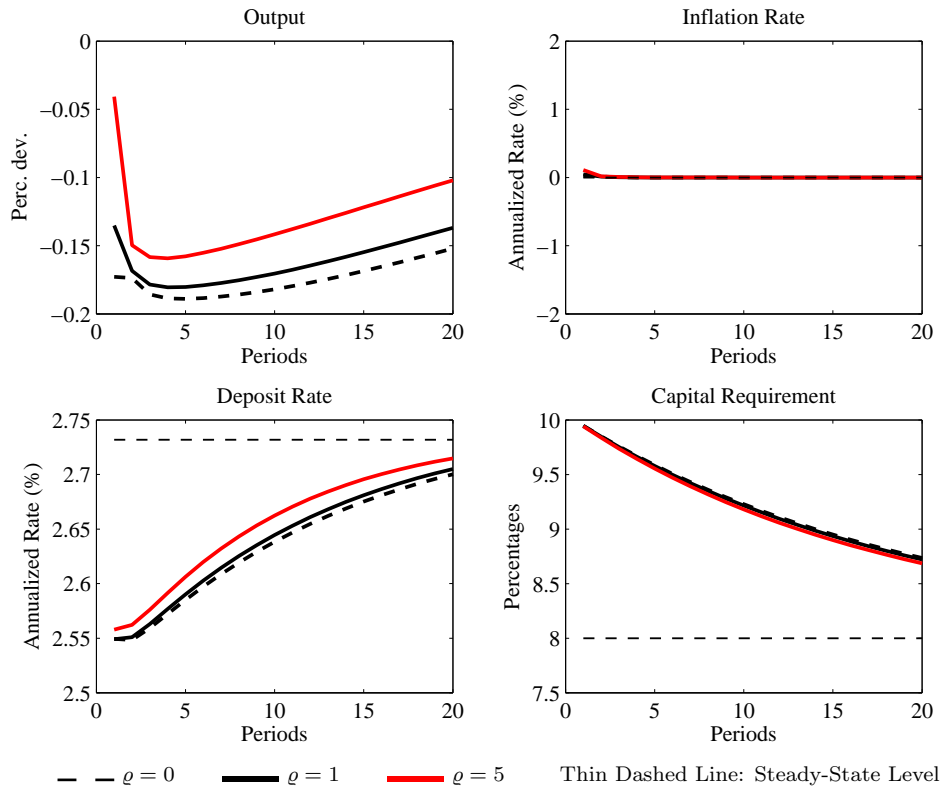


Figure 7: Response to an Increase in the Productivity of the Risky Technology ( $\eta_t^R$ )



## 6.2 Correlated shocks

Correlations across shocks may also link the risk-taking incentive to shocks that have direct business-cycle effects. As an example, we replace (7) by

$$k_{t+1}^S = \exp(\eta_t^S) x_t^S,$$

thus adding a shock to the safe technology for producing capital goods, and we allow for the possibility that  $\eta_t^S$  is correlated with  $\eta_t^R$  (the shock to the risky technology). This modification changes our solution for  $\kappa_t^*$  to

$$\kappa_t^* = (1 - \tau) \frac{(1 - \phi_t) \gamma_t [\exp(\eta_t^R - \eta_t^S) - 1] + \Psi_t [(1 - \phi_t) \gamma_t \exp(\eta_t^R - \eta_t^S) - \phi_t]}{\phi_t (1 + \gamma_t) - \gamma_t \tau (1 - \phi_t) [\exp(\eta_t^R - \eta_t^S) - 1]},$$

and changes (10) to

$$E_t \{\lambda_{t+1} q_{t+1}\} = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} (1 + R_t^S) \exp(-\eta_t^S) q_t^x.$$

Figure 8 is the analogue of Figure 2; it shows the optimal responses to a positive innovation in  $\eta_t^R$  for three values of its correlation with the innovation to  $\eta_t^S$ : 0.25, 0.50, and 0.75. The correlation makes both optimal policies act in a counter-cyclical way. Optimal prudential policy raises the capital requirement to tame risk taking, and optimal monetary policy raises the deposit rate to tame the inflationary effect of the investment boom. The counter-cyclical tendency of the policies is stronger when the correlation across shocks is higher.

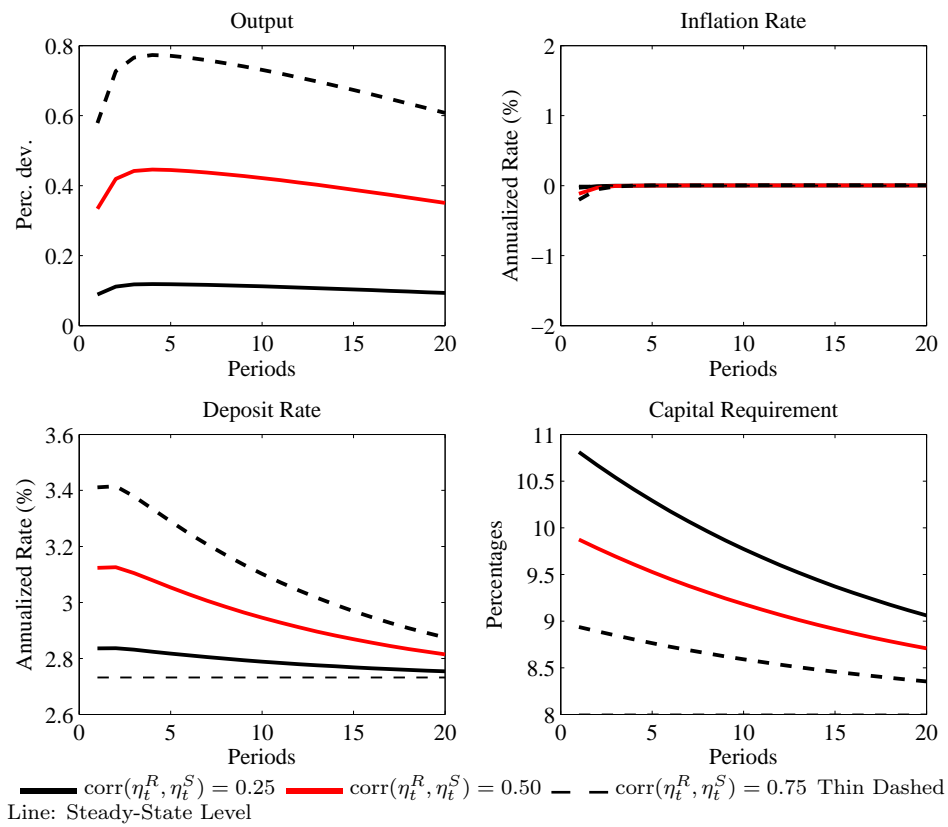
## 7 Concluding Remarks

The optimal interaction of monetary with prudential policy is a key issue in policy design that has not been fully addressed in the literature. In this paper we derive jointly optimal policies using a model that views bank capital requirements as a tool for addressing the risk-taking incentives created by limited liability and deposit insurance. Our model takes deposit insurance as an institutional feature that does not have to be rationalized within the model.<sup>25</sup> The other institutional feature is our assumption that a tax distortion makes equity finance more expensive than debt finance. We are not aware of any arguments for claiming that this is a feature of optimal policy in some expanded framework. To the contrary, existing discussions of this tax distortion [e.g., Admati et al. (2011), Mooij and Devereux (2011)] note its prevalence in OECD countries and call for removing it. Our motivation for including this policy-induced distortion in our model is its prevalence in reality and the fact that central banks and prudential regulators cannot change the tax code.<sup>26</sup> We think this tax distortion merits more attention in models of how the banking sector matters for monetary-policy

<sup>25</sup>Presenting an expanded model in which deposit insurance is optimal (rather than taking it as an exogenous feature) seemed too much of a digression to us, but we could motivate deposit insurance as usual [e.g., following Angeloni and Faia (2011)] in terms of ruling out equilibria with bank runs.

<sup>26</sup>Besides, under an arbitrarily small tax distortion, all our analytical results (from Proposition 1 to Proposition 6) still hold, as banks still prefer debt finance to equity finance, though the condition stated in Proposition 5 (the “if” part of this proposition) may not be met. In this case, our model is equivalent, at the first order, to a model in which the preference for debt finance would come from the presence of deposits in the utility function (as in Van den Heuvel, 2008) with an arbitrarily small weight.

Figure 8: Response to an Increase in the Productivity of the Risky Technology ( $\eta_t^R$ )



analysis.<sup>27</sup> In our model, it makes an increase in the capital requirement contractionary because it leads to higher bank lending rates.

Our benchmark model with perfectly competitive banks and constant marginal costs leads to a simple optimal assignment of tasks to prudential and monetary policies. The locally optimal mandate of prudential policy is to ensure that banks never fund inefficient risky projects, but to accomplish this objective with minimal damage in terms of increased bank lending rates and decreased capital stock. The distortion is minimized if capital requirements are state dependent. The interaction across policies then boils down to cutting (raising) interest rates to moderate the contractions (expansions) caused by changes in the capital requirement. The model also serves to illustrate how time variation in the capital requirement may be in response to shocks that affect the relative attractiveness of risky and safe projects.

The extension with an externality in the cost of banking, however, illustrates that optimal policy interactions may be more complex. In this example, an increase in the aggregate volume of safe loans increases the costs of originating and monitoring safe loans. This feature matters for the policy interactions. Compared to the model with no externality, the optimal expansion of output in response to a productivity shock is smaller. Moreover, because the optimal capital requirement rises in order to prevent excessive risk taking, the optimal monetary policy response does not fight the boom as much (and cuts the policy rate more aggressively later).

---

<sup>27</sup>For one thing, this may account for the fact that banks extend credit using loan contracts in reality, even though loan contracts are not optimal according to most formal models (with the notable exception of models with costly state verification).

## 8 Appendix

### 8.1 Proof of Proposition 1

To show that there is no equilibrium with  $0 < l_t^R < \gamma_t l_t^S$ , we suppose that there is such an equilibrium and consider a perturbation satisfying  $dl_t^S(j) = -dl_t^R(j)$  in the loan portfolio of a given bank  $j$ . Note that this perturbation neither tightens nor loosens bank  $j$ 's balance-sheet identity

$$l_t^S(j) + l_t^R(j) = e_t(j) + d_t(j) \quad (25)$$

and its capital requirement

$$e_t(j) \geq \kappa_t [l_t^S(j) + l_t^R(j)],$$

given that  $l_t^S(j) + l_t^R(j)$  is left unchanged. So this perturbation should not increase bank  $j$ 's expected excess return. The derivations of the effect of this perturbation on bank  $j$ 's expected excess return involves two cases, depending on whether firms' default leads to bank  $j$ 's default.

If firms' default leads to bank  $j$ 's default, then the change in bank  $j$ 's expected excess return is

$$(1 - \tau) \left[ \beta (1 - \phi_t) E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \frac{R_t^R - R_t^S}{\lambda_t} + \Psi_t \right] dl_t^R(j),$$

since bank  $j$  ignores the effect of its loan portfolio change on aggregate variables like  $\lambda_{t+1}$  or  $\Pi_{t+1}$ . As discussed in the main text, we must have  $R_t^R \geq R_t^S$  in equilibrium. Therefore, bank  $j$ 's expected excess return is increasing in  $l_t^R(j)$ . This means that bank  $j$  would like to take more risk, contradicting our conjecture about the existence of an equilibrium with  $l_t^R < \gamma_t l_t^S$ . This proves Part (a) of the Proposition.

If firms' default does not lead to bank  $j$ 's default, then the change in bank  $j$ 's expected excess return is

$$(1 - \tau) \left[ \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}} [\theta_t (1 + R_t^R) - (1 + R_t^S)] \right\} + \Psi_t \right] dl_t^R(j) \equiv M dl_t^R(j).$$

Now,

$$\begin{aligned} \frac{M}{1 - \tau} &= \beta (1 - \phi_t) \frac{(1 + R_t^R) - (1 + R_t^S)}{\lambda_t} E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \\ &\quad - \beta \phi_t \frac{1 + R_t^S}{\lambda_t} E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t \\ &= \beta (1 - \phi_t) \frac{1 + R_t^S}{\lambda_t} \left( \frac{1 + R_t^R}{1 + R_t^S} - 1 \right) E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \\ &\quad - \beta \phi_t \frac{1 + R_t^S}{\lambda_t} E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t \\ &\leq \beta (1 - \phi_t) \frac{1 + R_t^S}{\lambda_t} \left[ \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\}} \exp(\eta_t^R) - 1 \right] E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \right. \\ &\quad \left. \theta_t = 1 \right\} - \beta \phi_t \frac{1 + R_t^S}{\lambda_t} E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t \end{aligned}$$

where the last inequality comes from (12) and (17).

Therefore,

$$\begin{aligned} \frac{M}{1-\tau} &\leq \beta(1-\phi_t) \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} \exp(\eta_t^R) \\ &\quad - \beta(1-\phi_t) \frac{1+R_t^S}{\lambda_t} E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \\ &\quad - \beta\phi_t \frac{1+R_t^S}{\lambda_t} E_t \left\{ \left( \frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t, \end{aligned}$$

which implies

$$\frac{M}{1-\tau} \leq \beta(1-\phi_t) \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} \exp(\eta_t^R) - \beta \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} + \Psi_t$$

and

$$\frac{M}{1-\tau} \leq \beta \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} [(1-\phi_t) \exp(\eta_t^R) - 1] + \Psi_t.$$

Using (3), we get

$$\frac{M}{1-\tau} \leq \frac{1+R_t^S}{1+R_t^D} [(1-\phi_t) \exp(\eta_t^R) - 1] + \Psi_t.$$

and, using (8),

$$\frac{M}{1-\tau} \leq \Psi_t \left( 1 - \frac{1+R_t^S}{1+R_t^D} \right),$$

which implies  $M < 0$  because monitoring costs make  $R_t^S > R_t^D$  in equilibrium. Therefore, bank  $j$ 's expected excess return is decreasing in  $l_t^R(j)$ . This means that bank  $j$  would like to take less risk, contradicting our conjecture about the existence of an equilibrium with  $0 < l_t^R$ . This proves Part (b) of the Proposition.

## 8.2 Proof of Proposition 2

This appendix proves Proposition 2 by establishing a more general result that will serve us in subsequent appendices. We show that the capital constraint is always binding for a bank  $j$  that deviates from either the candidate equilibrium with  $l_t^R = 0$ , or the candidate equilibrium with  $l_t^R = \gamma_t l_t^S$  (the only two candidate equilibria left, given Proposition 1). As a consequence, for a zero deviation, the capital constraint is binding -i.e. (18) holds- in any of the two candidate equilibria of our model, which leads to Proposition 2.

In general, using (25), bank  $j$ 's expected excess return can be written

$$(1-\tau) E_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b(j)}{\lambda_t} \right\} - e_t(j) - (1-\tau) \Psi_t l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{R_t^S - R_t^D}{\Pi_{t+1}} l_t^S(j) + \left[ \theta_t \frac{1+R_t^R}{\Pi_{t+1}} - \frac{1+R_t^D}{\Pi_{t+1}} \right] l_t^R(j) + \frac{1+R_t^D}{\Pi_{t+1}} e_t(j) \right\}.$$

In the case where  $\omega_{t+1}^b(j) > 0$  when  $\theta_t = 0$ , using (3), bank  $j$ 's expected excess return can be rewritten

$$\begin{aligned} (1-\tau) \left\{ \frac{R_t^S - R_t^D}{1+R_t^D} l_t^S(j) + \left[ (1-\phi_t) \left( \frac{1+R_t^R}{1+R_t^D} \right) - 1 \right] l_t^R(j) + e_t(j) \right\} \\ - e_t(j) - (1-\tau) \Psi_t l_t^S(j). \end{aligned}$$

Since this expression is strictly decreasing in  $e_t(j)$ , it is maximized when  $e_t(j)$  is minimal, that is to say when  $e_t(j)$  satisfies

$$e_t(j) = \kappa_t [l_t^S(j) + l_t^R(j)]. \quad (26)$$

In the alternative case where  $\omega_{t+1}^b(j) = 0$  when  $\theta_t = 0$ , consider first the candidate equilibrium with  $l_t^R = 0$ . Bank  $j$ 's expected excess return can then be written

$$(1 - \tau)(1 - \phi_t) \left\{ \frac{R_t^S - R_t^D}{1 + R_t^D} l_t^S(j) + \left[ \frac{1 + R_t^R}{1 + R_t^D} - 1 \right] l_t^R(j) + e_t(j) \right\} - e_t(j) - (1 - \tau) \Psi_t l_t^S(j).$$

Since this expression is strictly decreasing in  $e_t(j)$ , it is maximized for  $e_t(j)$  given by (26). Consider next the candidate equilibrium with  $l_t^R = \gamma_t l_t^S$ . Bank  $j$ 's expected excess return can then be written

$$(1 - \tau)(1 - \phi_t) \frac{\beta}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} [(R_t^S - R_t^D) l_t^S(j) + (R_t^R - R_t^D) l_t^R(j) + (1 + R_t^D) e_t(j)] - e_t(j) - (1 - \tau) \Psi_t l_t^S(j).$$

This expression is strictly decreasing in  $e_t(j)$ , since its derivative with respect to  $e_t(j)$  is strictly negative:

$$\begin{aligned} & (1 - \tau)(1 - \phi_t) \frac{\beta}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} (1 + R_t^D) - 1 \\ = & (1 - \tau)(1 - \phi_t) \frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \exp(\eta_t^R) \frac{1 + R_t^S}{1 + R_t^R} - 1 \\ < & (1 - \tau)(1 - \phi_t) \exp(\eta_t^R) \frac{1 + R_t^S}{1 + R_t^R} - 1 \\ < & (1 - \tau)(1 - \Psi_t) \frac{1 + R_t^S}{1 + R_t^R} - 1 \\ < & 0, \end{aligned}$$

where the equality comes from (12) and (3), the first inequality from (17), the second inequality from (8), and the third inequality from the fact that  $R_t^R > R_t^S$  in equilibrium. Therefore, bank  $j$  will choose the minimal capital requirement, i.e.  $e_t(j)$  satisfying (26). To sum up, the capital constraint is always binding for a bank  $j$  that deviates from either the candidate equilibrium with  $l_t^R = 0$ , or the candidate equilibrium with  $l_t^R = \gamma_t l_t^S$ . In particular, for a zero deviation, the capital constraint is binding –i.e. (18) holds– in any of the two candidate equilibria of our model. This establishes Proposition 2.

### 8.3 Proof of Proposition 3

Consider a bank  $j$  that takes the maximum amount of risk by setting  $l_t^R(j) = \gamma_t l_t^S(j)$ . Using (25) and (26) to eliminate  $d_t(j)$  from

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(j) + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(j) - \frac{1 + R_t^D}{\Pi_{t+1}} d_t(j) \right\},$$

it is straightforward to show that this bank remains solvent ( $\omega_{t+1}^b(j) > 0$ ) when risky projects fail ( $\theta_t = 0$ ) if and only if (20) holds. Part (a) of Proposition 3 follows.



Then, consider a candidate equilibrium with  $l_t^R = 0$ . Using (13) to eliminate  $d_t$  and (18) to eliminate  $e_t$ , the representative bank's expected excess return can be rewritten

$$(1 - \tau) E_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b}{\lambda_t} \right\} - [\kappa_t + (1 - \tau) \Psi_t] l_t^S,$$

where

$$\omega_{t+1}^b = \left[ \frac{R_t^S - R_t^D}{\Pi_{t+1}} + \frac{1 + R_t^D}{\Pi_{t+1}} \kappa_t \right] l_t^S.$$

The representative bank chooses  $l_t^S$  so as to maximize its expected excess return. Using (3), the first-order condition of this programme can be written

$$(1 - \tau) \frac{R_t^S - R_t^D}{1 + R_t^D} - \tau \kappa_t - (1 - \tau) \Psi_t = 0. \quad (27)$$

We can then use this first-order condition to rewrite  $\tilde{\kappa}(R_t^D, R_t^S)$ , at the candidate equilibrium with  $l_t^R = 0$ , as (19). Parts (b) and (c) of Proposition 3 follow.

#### 8.4 Proof of Proposition 4

To prove Part (a) of Proposition 4, we look for a necessary and sufficient condition on policy instruments for the existence of an equilibrium with  $l_t^R = 0$ . This amounts to looking for a necessary and sufficient condition on policy instruments for the demand and supply curves on the risky-loans market to intersect at one or several points  $(R_t^R, l_t^R)$  with  $R_t^R \geq 0$  and  $l_t^R = 0$ . We proceed in several steps.

**Step 1: condition for zero demand for risky loans.** Given capital producers' programme, the portion of the demand curve that is consistent with  $l_t^R = 0$  is characterized by

$$\frac{1 + R_t^R}{1 + R_t^S} \geq \frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} | \theta_t = 1 \right\}} \exp(\eta_t^R).$$

Because  $\theta_t$  is independent of any other shock and because the realization of  $\theta_t$  does not affect the aggregate outcome when  $l_t^R = 0$ , the latter inequality can be rewritten

$$\frac{1 + R_t^R}{1 + R_t^S} \geq \exp(\eta_t^R). \quad (28)$$

**Step 2: condition for zero supply of risky loans.** The portion of the supply curve that is consistent with  $l_t^R = 0$  can be characterized by a necessary and sufficient condition for an individual bank  $j$  not to deviate from the candidate equilibrium with  $l_t^R = 0$ . We now look for such a condition. Appendix 8.1 implies that, if some deviations are profitable, then the most profitable deviation is  $l_t^R(j) = \gamma_t l_t^S(j)$ . If bank  $j$  makes this deviation, then, using (25) to eliminate  $d_t(j)$  and (26) to eliminate  $e_t(j)$ , its expected excess return can be rewritten

$$(1 - \tau) E_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b(j)}{\lambda_t} \right\} - [\kappa_t (1 + \gamma_t) + (1 - \tau) \Psi_t] l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \left[ \frac{R_t^S - R_t^D}{\Pi_{t+1}} + \theta_t \gamma_t \frac{1 + R_t^R}{\Pi_{t+1}} - \gamma_t \frac{1 + R_t^D}{\Pi_{t+1}} + \frac{1 + R_t^D}{\Pi_{t+1}} \kappa_t (1 + \gamma_t) \right] l_t^S(j) \right\}.$$

Because  $\theta_t$  is independent of any other shock and because the realization of  $\theta_t$  does not affect the aggregate outcome in equilibrium (given that  $l_t^R = 0$ ), bank  $j$ 's expected excess return can be rewritten, using (3),

$$(1 - \tau) E_t \left\{ \max \left\{ 0, \left[ \frac{R_t^S - R_t^D}{1 + R_t^D} + \theta_t \gamma_t \frac{1 + R_t^R}{1 + R_t^D} - \gamma_t + \kappa_t (1 + \gamma_t) \right] l_t^S(j) \right\} \right\} - [\kappa_t (1 + \gamma_t) + (1 - \tau) \Psi_t] l_t^S(j).$$

Note that the ‘max’ that features in this expression is strictly higher than zero when  $\theta_t = 1$ , because both  $R_t^R$  and  $R_t^S$  are strictly higher than  $R_t^D$  in equilibrium. So we will have to consider two cases, depending on whether this ‘max’ is strictly higher than zero or equal to zero when  $\theta_t = 0$ .

In the case where this ‘max’ is strictly higher than zero when  $\theta_t = 0$ , that is to say in the case where  $\kappa_t > \tilde{\kappa}_t$ , we know from Proposition 1 that bank  $j$ 's deviation is not profitable.

In the alternative case where the ‘max’ is equal to zero when  $\theta_t = 0$ , that is to say in the case where  $\kappa_t \leq \tilde{\kappa}_t$ , bank  $j$ 's expected excess return is

$$\left\{ (1 - \tau) (1 - \phi_t) \left[ \frac{R_t^S - R_t^D}{1 + R_t^D} + \gamma_t \frac{1 + R_t^R}{1 + R_t^D} - \gamma_t + \kappa_t (1 + \gamma_t) \right] - \kappa_t (1 + \gamma_t) - (1 - \tau) \Psi_t \right\} l_t^S(j).$$

Using (27) to eliminate  $R_t^S$ , we can then rewrite bank  $j$ 's expected excess return as

$$\left\{ (1 - \tau) (1 - \phi_t) \gamma_t \frac{R_t^R - R_t^D}{1 + R_t^D} - [\phi_t (1 + \gamma_t) + \gamma_t \tau (1 - \phi_t)] \kappa_t - \phi_t (1 - \tau) \Psi_t \right\} l_t^S(j).$$

Therefore, a necessary and sufficient condition for the deviation not to be profitable is then

$$[\phi_t (1 + \gamma_t) + \gamma_t \tau (1 - \phi_t)] \kappa_t + \phi_t (1 - \tau) \Psi_t \geq (1 - \tau) (1 - \phi_t) \gamma_t \frac{R_t^R - R_t^D}{1 + R_t^D}. \quad (29)$$

To sum up, the portion of the supply curve that is consistent with  $l_t^R = 0$  is characterized by the condition that either  $\kappa_t > \tilde{\kappa}_t$ , or  $\kappa_t \leq \tilde{\kappa}_t$  and (29) holds.

**Step 3: condition for zero risky loans in equilibrium.** The demand and supply curves on the risky-loans market intersect at one or several points  $(R_t^R, l_t^R)$  with  $R_t^R \geq 0$  and  $l_t^R = 0$  if and only if either (i)  $\kappa_t > \tilde{\kappa}_t$ , or (ii)  $\kappa_t \leq \tilde{\kappa}_t$ , and (29) holds when (28) holds with equality.

Note that, if (28) holds with equality, then, using (27), we can rewrite (29) as

$$\kappa_t \geq \kappa_t^* \equiv (1 - \tau) \frac{(1 - \phi_t) \gamma_t [\exp(\eta_t^R) - 1] + \Psi_t [(1 - \phi_t) \gamma_t \exp(\eta_t^R) - \phi_t]}{\phi_t (1 + \gamma_t) - \gamma_t \tau (1 - \phi_t) [\exp(\eta_t^R) - 1]}, \quad (30)$$

since the denominator on the right-hand side of this inequality is strictly positive:

$$\begin{aligned} & \phi_t (1 + \gamma_t) - \gamma_t \tau (1 - \phi_t) [\exp(\eta_t^R) - 1] \\ &= \phi_t [1 + \gamma_t (1 - \tau)] + \gamma_t \tau - \gamma_t \tau (1 - \phi_t) \exp(\eta_t^R) \\ &> \phi_t [1 + \gamma_t (1 - \tau)] + \gamma_t \tau \Psi_t \\ &> 0, \end{aligned}$$

where the last but one inequality comes from (8). As a consequence, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with  $l_t^R = 0$  is that either  $\kappa_t > \tilde{\kappa}_t$ ,

or  $\kappa_t^* \leq \kappa_t \leq \tilde{\kappa}_t$ . This condition can be equivalently rewritten  $\kappa_t \geq \min\{\tilde{\kappa}_t, \kappa_t^*\}$ . Now, using (8) to replace  $(1 - \phi_t) \exp(\eta_t^R)$  by  $1 - \Psi_t$  on the right-hand side of (30), we get

$$\begin{aligned} \kappa_t^* &\leq (1 - \tau) \frac{-\gamma_t \Psi_t^2 + \phi_t (\gamma_t - \Psi_t)}{\gamma_t \tau \Psi_t + \phi_t (1 + \gamma_t - \gamma_t \tau)} \\ &= \tilde{\kappa}_t \left\{ 1 - \frac{\gamma_t \Psi_t}{\gamma_t - \Psi_t} \frac{\gamma_t \tau + \Psi_t (1 + \gamma_t) (1 - \tau)}{\gamma_t \Psi_t \tau + \phi_t (1 + \gamma_t - \gamma_t \tau)} \right\} \\ &< \tilde{\kappa}_t, \end{aligned}$$

where the last inequality comes from our assumption that  $\gamma_t > \Psi_t$ . Therefore, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with  $l_t^R = 0$  is simply  $\kappa_t \geq \kappa_t^*$ . Parts (a) and (b) of Proposition 4 follow.

Finally, Part (c) of Proposition 4 follows straightforwardly from the fact that the denominator on the right-hand side of (30) is strictly positive, as shown above.

## 8.5 Proof of Proposition 5

Define welfare as the representative household's expected utility at date 0,  $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$ . For any policy  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$ , define the distance from  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  as

$$\varepsilon \equiv \max \left[ \max_{\tau \geq 0} (|R_\tau^D - R_\tau^{D*}|), \max_{\tau \geq 0} (|\kappa_\tau - \kappa_\tau^*|) \right].$$

Let us first compare  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  to policies  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$  such that  $\varepsilon$  is arbitrarily small and  $\exists t \geq 0$ ,  $\kappa_t < \kappa_t^*$ . Moving from  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  to any such policy triggers a discontinuous increase in the amount of risk, as it makes banks' risky loans  $l_t^R$  move from 0 to  $\gamma_t l_t^S > 0$  at some date  $t \geq 0$ . Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between  $\theta_t$  and other shocks), this discontinuous increase in the amount of risk has a discontinuous negative effect on welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that  $\varepsilon$  is small enough. As a consequence, welfare is strictly higher under  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  than under any such policy provided that  $\varepsilon$  is small enough.

Let us then compare  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  to policies  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$  such that  $\varepsilon$  is arbitrarily small,  $\forall \tau \geq 0$ ,  $\kappa_\tau \geq \kappa_\tau^*$ , and  $\exists t \geq 0$ ,  $\kappa_t > \kappa_t^*$ . Using the equilibrium conditions that are independent of policies, rewrite welfare as

$$W \left[ (R_\tau^D)_{\tau \geq 0}, (\kappa_\tau)_{\tau \geq 0}, H_0 \right],$$

where  $H_0$  captures initial conditions (endogenous variables until date  $-1$ , exogenous shocks until date 0). Since  $(R_\tau^{D*})_{\tau \geq 0}$  is the monetary policy that is Ramsey-optimal when  $(\kappa_\tau)_{\tau \geq 0} = (\kappa_\tau^*)_{\tau \geq 0}$ , we have

$$\forall t \geq 0, \frac{\partial W}{\partial R_t^D} \left[ (R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] = 0.$$

Therefore, the first-order Taylor approximation of  $W \left[ (R_\tau^D)_{\tau \geq 0}, (\kappa_\tau)_{\tau \geq 0}, H_0 \right]$  in a neighborhood of

$\left[ (R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right]$  such that  $\forall \tau \geq 0, \kappa_\tau \geq \kappa_\tau^*$ , is

$$W \left[ (R_\tau^D)_{\tau \geq 0}, (\kappa_\tau)_{\tau \geq 0}, H_0 \right] = W \left[ (R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] + \sum_{t=0}^{+\infty} \frac{\partial W}{\partial \kappa_t} \left[ (R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] (\kappa_t - \kappa_t^*) + O(\varepsilon^2),$$

where  $\frac{\partial W}{\partial \kappa_t}$  is the right derivative of welfare with respect to  $\kappa_t$  and  $O(\varepsilon^2)$  is a term of second order in  $\varepsilon$ . As a consequence, if

$$\forall t \geq 0, \frac{\partial W}{\partial \kappa_t} \left[ (R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] < 0,$$

then welfare is strictly higher under  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  than under any policy  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$  such that  $\forall \tau \geq 0, \kappa_\tau \geq \kappa_\tau^*$  and  $\exists t \geq 0, \kappa_t > \kappa_t^*$ , provided that  $\varepsilon$  is small enough. Proposition 5 follows.

## 8.6 Proof of Proposition 6

Using (27) and (28), it is easy to show that, at any candidate equilibrium with  $l_t^R = 0$ , the prudential-policy rule (23) implies (i)  $\kappa_t \geq \kappa_t^*$  and (ii)  $\kappa_t = \kappa_t^*$  if and only if (28) holds with equality. Therefore, given Proposition 4, there exists a unique equilibrium with  $l_t^R = 0$  under (23) and, at this equilibrium,  $\kappa_t = \kappa_t^*$  and (28) holds with equality.

We now show that there exists no equilibrium with  $l_t^R = \gamma_t l_t^S$  under (23). To that aim, consider a candidate equilibrium with  $l_t^R = \gamma_t l_t^S$ . Proposition 1 implies that, if  $\omega_{t+1}^b > 0$  when  $\theta_t = 0$ , then this candidate equilibrium is not an equilibrium. We focus therefore on the case where  $\omega_{t+1}^b = 0$  when  $\theta_t = 0$ . Consider a given bank  $j$ , whose expected excess return is

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1 - \tau) \omega_{t+1}^b(j)}{\lambda_t} \right\} - e_t(j) - (1 - \tau) \Psi_t l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(j) + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(j) - \frac{1 + R_t^D}{\Pi_{t+1}} d_t(j) \right\}.$$

Using (25) to eliminate  $d_t(j)$  and (26) to eliminate  $e_t(j)$ , its expected excess return can be rewritten

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1 - \tau) \omega_{t+1}^b(j)}{\lambda_t} \right\} - \kappa_t [l_t^S(j) + l_t^R(j)] - (1 - \tau) \Psi_t l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(j) + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(j) - \frac{1 + R_t^D}{\Pi_{t+1}} (1 - \kappa_t) [l_t^S(j) + l_t^R(j)] \right\}.$$

If bank  $j$  does not deviate from the candidate equilibrium with  $l_t^R = \gamma_t l_t^S$ , then its expected excess return is equal to

$$\left[ (1 - \phi_t) \beta \frac{(1 - \tau) [(1 + R_t^S) + \gamma_t (1 + R_t^R) - (1 + \gamma_t) (1 - \kappa_t) (1 + R_t^D)]}{\lambda_t} \right. \\ \left. E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - \kappa_t (1 + \gamma_t) - (1 - \tau) \Psi_t \right] \frac{1}{1 + \gamma_t} l_t(j),$$

where  $l_t(j) \equiv l_t^S(j) + l_t^R(j)$ , since  $\omega_{t+1}^b(j) = 0$  when  $\theta_t = 0$ . Appendix 8.1 implies that, if some deviations from the candidate equilibrium with  $l_t^R = \gamma_t l_t^S$  are profitable, then the most profitable deviation is to provide zero risky loans. If bank  $j$  makes this deviation, then its expected excess return becomes

$$\left[ \phi_t \beta \frac{(1-\tau) [(R_t^S - R_t^D) + \kappa_t (1 + R_t^D)]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\} \right. \\ \left. + (1 - \phi_t) \beta \frac{(1-\tau) [(R_t^S - R_t^D) + \kappa_t (1 + R_t^D)]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - \kappa_t - (1 - \tau) \Psi_t \right] l_t(j).$$

The change in bank  $j$ 's expected excess return, from  $l_t^R(j) = \gamma_t l_t^S(j)$  to  $l_t^R(j) = 0$ , is

$$\left[ \phi_t \beta \frac{(1-\tau) [(R_t^S - R_t^D) + \kappa_t (1 + R_t^D)]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\} \right. \\ \left. - (1 - \phi_t) \beta \frac{(1-\tau)}{\lambda_t} \frac{\gamma_t}{1 + \gamma_t} (R_t^R - R_t^S) E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - (1 - \tau) \frac{\gamma_t}{1 + \gamma_t} \Psi_t \right] l_t(j).$$

It is easy to show that this change is strictly positive, and therefore that bank  $j$  gains from deviating from the candidate equilibrium with  $l_t^R = \gamma_t l_t^S$ , if and only if

$$\kappa_t > -\frac{R_t^S - R_t^D}{1 + R_t^D} + \frac{\gamma_t}{1 + \gamma_t} \frac{(1 - \phi_t) \beta \frac{R_t^R - R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} + \Psi_t}{\phi_t \beta \frac{(1 + R_t^D)}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\}} \\ \equiv \hat{\kappa} \left( R_t^D, R_t^S, R_t^R, \lambda_t, \frac{\lambda_{t+1}}{\Pi_{t+1}}, \phi_t \right).$$

Therefore, there exists no equilibrium with  $l_t^R = \gamma_t l_t^S$  under the prudential-policy rule (23) if

$$\kappa^* (R_t^D, R_t^S, R_t^R, \phi_t) > \hat{\kappa} \left( R_t^D, R_t^S, R_t^R, \lambda_t, \frac{\lambda_{t+1}}{\Pi_{t+1}}, \phi_t \right),$$

where  $\kappa^* (R_t^D, R_t^S, R_t^R, \phi_t)$  is the expression on the right-hand side of (23). Using (3), the latter inequality is easily shown to be equivalent to

$$\frac{1 - \phi_t}{\phi_t^2} \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\}} \left[ 1 - \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}} \right] \left( \frac{R_t^R - R_t^S}{1 + R_t^D} + \Psi_t \right) > 0$$

and is therefore satisfied, given (22) and  $R_t^R \geq R_t^S$ . This establishes Proposition 6.

## References

- [1] Admati, Anat R., Peter M. DeMarzo, Martin F. Hellwig, and Paul Pfleiderer (2011), “Fallacies, Irrelevant Facts, and Myths in Capital Regulation: Why Bank Equity is Not Expensive,” mimeo.
- [2] Angeloni, Ignazio, and Ester Faia (2011), “Capital Regulation and Monetary Policy with Fragile Banks,” mimeo.
- [3] Basel Committee on Banking Supervision (2010), “Basel III: A global regulatory framework for more resilient banks and banking systems,” Bank for International Settlements.
- [4] Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric Young (2011), “Monetary and Macro-Prudential Policies: An Integrated Analysis,” mimeo.
- [5] Bernanke, Ben (2010), “Monetary Policy and the Housing Bubble,” Remarks at the American Economic Association meetings, Atlanta, GA, January 3.
- [6] Bianchi, Javier (2011), “Overborrowing and Systemic Externalities in the Business Cycle,” *American Economic Review*, 101(7): 3400-3426.
- [7] Bianchi, Javier, and Enrique Mendoza (2011), “Overborrowing, Financial Crises and ‘Macroprudential’ Policy,” IMF Working Paper 11/24.
- [8] Blanchard, Olivier, Giovanni Dell’Ariccia, and Paolo Mauro (2010), “Rethinking Macroeconomic Policy,” *Journal of Money, Credit and Banking*, 42: 199-215.
- [9] Borio, Claudio, and Haibin Zhu (2008), “Capital Regulation, Risk-Taking and Monetary Policy: A Missing Link in the Transmission Mechanism?,” BIS Working Paper No. 268.
- [10] Canuto, Otaviano (2011), “How Complementary Are Prudential Regulation and Monetary Policy?,” *Economic Premise* No. 60.
- [11] Cecchetti, Stephen G., and Marion Kohler (2012), “When Capital Adequacy and Interest Rate Policy Are Substitutes (and When They Are Not),” BIS Working Paper No. 379.
- [12] Christensen, Ian, Cesaire Meh, and Kevin Moran (2010) “Bank Leverage Regulation and Macroeconomic Dynamics,” mimeo.
- [13] Covas, Francisco, and Shigeru Fujita (2010), “Procyclicality of Capital Requirements in a General Equilibrium Model of Liquidity Dependence,” *International Journal of Central Banking*, 6(4): 137-173.
- [14] Cúrdia, Vasco, and Michael Woodford (2009), “Conventional and Unconventional Monetary Policy,” mimeo.
- [15] Freixas, Xavier, and Jean-Charles Rochet (2008), *Microeconomics of Banking*, Second Edition, Cambridge, MA: MIT Press.
- [16] Gete, Pedro, and Natalie Tiernan (2011), “Lax Lending Standards and Capital Requirements,” mimeo.

- [17] Hachem, Kinda (2010), “Screening, Lending Intensity, and the Aggregate Response to a Bank Tax,” mimeo.
- [18] Jeanne, Olivier, and Anton Korinek (2010), “Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach,” *American Economic Review*, 100(2): 403-407.
- [19] Jermann, Urban, and Vincenzo Quadrini (2009), “Financial Innovations and Macroeconomic Volatility,” mimeo.
- [20] Jermann, Urban, and Vincenzo Quadrini (2012), “Macroeconomic Effects of Financial Shocks,” *American Economic Review*, 102(1): 238-271.
- [21] Kashyap, Anil, Richard Berner, and Charles Goodhart (2011), “The Macroprudential Toolkit,” Chicago Booth Working Paper No. 11-02.
- [22] Levin, Andrew, and David López-Salido (2004), “Optimal Monetary Policy with Endogenous Capital Accumulation,” mimeo.
- [23] Levin, Andrew, Alexei Onatski, John Williams, and Noah Williams (2005), “Monetary Policy under Uncertainty in Microfounded Macroeconometric Models,” in: *NBER Macroeconomics Annual 2005*, Gertler, M., Rogoff, K., eds, Cambridge, MA: MIT Press.
- [24] Macklem, Tiff (2011), “Mitigating Systemic Risk and the Role of Central Banks,” speech to Conférence de Montréal, Montréal, Québec, June 6.
- [25] Martinez-Miera, David, and Javier Suarez (2012), “A Macroeconomic Model of Endogenous Systemic Risk Taking,” mimeo.
- [26] Mooij, Ruud, and Michael Devereux (2011), “An Applied Analysis of ACE and CBIT Reforms in the EU,” *International Tax and Public Finance*, 18(1): 93-120.
- [27] Schularick, Moritz, and Alan M. Taylor (2012), “Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008,” *American Economic Review*, 102(2): 1029-1061.
- [28] Van den Heuvel, Skander J. (2008), “The Welfare Cost of Bank Capital Requirements,” *Journal of Monetary Economics*, 55: 298-320.
- [29] Wolf, Martin (2012), “After the Bonfire of the Verities,” *Financial Times*, May 1.
- [30] Yellen, Janet (2010), “Macroprudential Supervision and Monetary Policy in the Post-Crisis World,” speech at the Annual Meeting of the National Association for Business Economics, Denver, CO, October 11.

## Documents de Travail

396. M. Bussiere and A. Ristinieni, "Credit Ratings and Debt Crises," September 2012
397. A. Monfort and F. Pegoraro, "Asset Pricing with Second-Order Esscher Transforms," September 2012
398. S. Gabrieli, "Too-connected versus too-big-to-fail: banks' network centrality and overnight interest rate," September 2012
399. Y. Kalantzis, R. Kambayashi and S. Lechevalier, "Wage and Productivity differentials in Japan. The Role of Labor Market Mechanisms," September 2012
400. F. Bec and M. Bessec, "Inventory Investment Dynamics and Recoveries: A Comparison of Manufacturing and Retail Trade Sectors," October 2012
401. P. Antipa, K. Barhouni, V. Brunhes-Lesage and O. Darné, "Nowcasting German GDP: A comparison of bridge and factor models," October 2012
402. G. Gaballo, "Good Luck or Good Policy? An Expectational Theory of Macro-Volatility Switches," October 2012
403. J. Barthélemy and M. Marx, "Generalizing the Taylor Principle: New Comment," October 2012
404. H. Fraise and P. Frouté, "Households Debt Restructuring: Evidence from the French Experience," October 2012
405. E. Kremp and P. Sevestre, "Did the crisis induce credit rationing for French SMEs?," November 2012
406. P. Bacchetta, K. Benhima and Y. Kalantzis, "Capital Controls with International Reserve Accumulation: Can this Be Optimal?," November 2012
407. P. Andrade, E. Ghysels and J. Idier, "Tails of Inflation Forecasts and Tales of Monetary Policy," November 2012
408. N. Berman, A. Berthou and J. Héricourt, "Export dynamics and sales at home," November 2012
409. M. Bessec, "Short-term forecasts of French GDP: a dynamic factor model with targeted predictors," November 2012
410. J. Carluccio and T. Fally, "Foreign Entry and Spillovers with Technological Incompatibilities in the Supply Chain," November 2012
411. T. Duprey, "Bank Ownership and Credit Cycle: the lower sensitivity of public bank lending to the business cycle," December 2012
412. O. Loisel, A. Pommeret and F. Portier, "Monetary Policy and Herd Behavior: Leaning Against Bubbles," December 2012
413. F. Collard, H. Dellas, B. Diba and O. Loisel, "Optimal Monetary and Prudential Policies," December 2012

Pour accéder à la liste complète des Documents de Travail publiés par la Banque de France veuillez consulter le site : [www.banque-france.fr](http://www.banque-france.fr)

For a complete list of Working Papers published by the Banque de France, please visit the website: [www.banque-france.fr](http://www.banque-france.fr)

Pour tous commentaires ou demandes sur les Documents de Travail, contacter la bibliothèque de la Direction Générale des Études et des Relations Internationales à l'adresse suivante :

For any comment or enquiries on the Working Papers, contact the library of the Directorate General Economics and International Relations at the following address :

BANQUE DE FRANCE  
49- 1404 Labolog  
75049 Paris Cedex 01  
tél : 0033 (0)1 42 97 77 24 ou 01 42 92 63 40 ou 48 90 ou 69 81  
email : [1404-ut@banque-france.fr](mailto:1404-ut@banque-france.fr)