NOTES D'ÉTUDES

ET DE RECHERCHE

ML VS GMM ESTIMATES OF HYBRID MACROECONOMIC MODELS (WITH AN APPLICATION TO THE "NEW PHILLIPS CURVE")

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October 2003

NER # 103

BANQUE DE FRANCE

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

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(First draft: February 2003)

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This paper has been presented at the ESEM in August 2003, Stockholm, and at seminars at the Banque de France, the ECB ("Inflation Persistence Network"), ERUDITE (Université Paris 12 Val de Marne), GREQAM (Université de la Méditerranée), and CORE (Catholic University of Louvain). We benefited from comments of and discussions with Russell Davidson, Patrick Fève, André Kurmann, François Legendre, Julien Matheron, Sophocles Mavroeidis, and Patrick Sevestre. This paper does not necessarily reflect the views of the Banque de France.

Résumé :

De nombreux modèles macroéconomiques incorporent des équations "hybrides", dans lesquelles certaines variables sont fonctions à la fois de leurs valeurs retardées et de leur anticipation. La courbe de Phillips hybride de la "nouvelle économie keynésienne" en constitue une illustration. Les estimations de ce type de modèles ont produit des résultats empiriques contrastés : les études qui utilisent des estimations par maximum de vraisemblance (MV) tendent à trouver une composante retardée dominante, alors que les études mettant en œuvre la méthode des moments généralisée (MMG) trouvent que la dynamique de l'inflation est essentiellement tournée vers l'avenir.
Ce papier propose une explication à ces résultats conflictuels. En introduisant deux types d'erreur de spécification standard (erreur de mesure et dynamique manquante), nous montrons que l'estimateur MV tend à sous-évaluer le poids de la composante anticipée, alors que l'estimateur MMG tend à la sur-évaluer. Nous obtenons ce résultat de façon analytique dans un modèle simple. Des simulations de Monte-Carlo indiquent que l'ordonnancement des estimateurs demeure valide pour une large étendue de modèles. A l'aide de simulations, nous trouvons que l'écart obtenu entre les deux estimateurs MMG et MV dans le contexte de la nouvelle courbe de Phillips peut s'expliquer par une erreur de spécification plus facilement que par des biais de petit échantillon parfois invoqués.

Mots-clés : Modèles à anticipations rationnelles, Méthode des moments généralisée, Maximum de vraisemblance, Inflation, Nouvelle courbe de Phillips.

Abstract:

Many macroeconomic models involve hybrid equations, in which some variables are a function of both their lags and their expected future value. The hybrid "New Keynesian" Phillips Curve is a prominent example. Estimates of such hybrid models have produced conflicting empirical results: Studies which use ML estimation tend to find the forward-looking component to be small, while those using GMM have reported the inflation dynamics to be predominantly forward-looking.
This paper provides a rationalization for this empirical conflict. Allowing for two alternative and straightforward mis-specifications (measurement error and omitted dynamics) in a hybrid model, we show that the ML estimator tends to undervalue the weight of the forward-looking component, while the GMM estimator tends to overstate it. This result is shown to hold analytically in a simple DGP. Monte-Carlo experiments indicate that it remains valid in a wide range of more plausible DGPs. Simulations also suggest that the gap obtained between the two estimators in the context of the new Phillips curve can more readily be accounted for by mis-specification, than by the finite-sample biases.

Keywords: Rational-expectation model, GMM estimator, ML estimator, Inflation, New Phillips curve.

JEL classification: C10, C22, E31.

Résumé non technique :

De nombreux modèles macroéconomiques incorporent des équations "hybrides", dans lesquelles certaines variables sont fonctions à la fois de leurs valeurs retardées et de leur valeur future anticipée. L'appellation hybride renvoie au fait qu'un tel modèle englobe une spécification uniquement fondée sur des termes d'anticipations, telle qu'elle peut être dérivée par la théorie économique – typiquement au travers d'une équation d'Euler –, et une spécification plus empirique, du type autorégressive à retards échelonnés. La courbe de Phillips hybride de la "nouvelle économie keynésienne" constitue une illustration notable de ce type de spécifications. L'intérêt de cette courbe de Phillips hybride, proposée par Fuhrer et Moore (1995), Galí et Gertler (1999) ou Christiano, Eichenbaum, et Evans (2001), est de rendre compte de l'inertie de l'inflation, non prédite par les modèles théoriques sous-jacents à la "nouvelle courbe de Phillips" (Taylor, 1980, Rotemberg, 1982 ou Calvo, 1983) dans laquelle la fixation des prix est purement tournée vers le futur.

Sous l'hypothèse d'anticipations rationnelles, l'estimation de modèles de type hybride est généralement menée en utilisant soit la méthode du maximum de vraisemblance (MV), soit la méthode de moments généralisée (MMG). A ces deux méthodes sont associées deux manières de prendre en compte la présence dans le modèle de la variable d'anticipation, qui n'est pas observée par l'économètre. La MMG repose sur la projection de la valeur future de la variable d'inflation (observée *ex post* par l'économètre) sur un ensemble potentiellement large d'instruments, sans spécifier la loi d'évolution des variables explicative. La méthode du ML repose sur la spécification d'un modèle complet décrivant la dynamique propre de variables explicatives, et utilise la résolution de ce modèle pour contruire une valeur implicite d'inflation anticipée cohérente avec le modèle. Or, les estimations de courbes de Phillips hybrides ont à ce jour produit des résultats empiriques contrastés, en particulier en ce qui concerne le poids relatif des composantes anticipée et inertielle dans la détermination du taux d'inflation. Les études qui utilisent l'estimation par MV tendent à trouver une composante retardée (inertielle) dominante, alors que les études mettant en œuvre la MMG trouvent que la dynamique de l'inflation est essentiellement influencée par les anticipations.

Le présent article propose une explication à ces résultats conflictuels. Tout d'abord, nous soulignons à l'aide de simulations que, sauf à introduire un nombre très important de variables instrumentales redondantes, les biais de petit échantillon des estimateurs ne peuvent conduire à une différence subtantielle dans l'estimation du poids de la composante anticipée de l'inflation. En revanche, l'introduction de deux types d'erreur de spécification standard (erreur de mesure et dynamique manquante) nous permet de d'obtenir une différence importante entre les estimateurs obtenus par les deux méthodes. Il est notamment montré que l'estimateur MV tend à sous-évaluer le poids de la composante anticipée, alors que l'estimateur MMG tend à la sur-évaluer. Ce résultat est obtenu de façon analytique dans un modèle simple. Des simulations de Monte-Carlo indiquent qu'un tel ordonnancement des estimateurs MMG et MV demeure valide pour une large étendue de valeurs des paramètres.

Les résultats obtenus conduisent à mettre l'accent sur une configuration particulière dans laquelle le biais affectant les estimateurs MV et MMG est de sens opposé. Il s'agit du cas où une variable explicative est omise de l'équation d'intérêt, tout en étant introduite dans le jeu d'instruments de la MMG. Dans un tel cas, la valeur "prévue" de la variable endogène future,

implicitement utilisée dans l'équation d'intérêt par la MMG, et obtenue par projection de la valeur future réalisée sur les différents instruments, est une fonction de la variable omise. Dès lors, cette valeur "prévue" tend à capturer l'effet de la variable omise sur la variable d'intérêt, et le paramètre associé à la composante anticipée est sur-évalué.

Dans la partie empirique de notre travail, nous utilisons les résultats obtenus afin de revisiter les estimations de la courbe de Phillips hybride sur données américaines. Un écart entre les deux méthodes apparaît bien quant à l'estimation du poids de la composante anticipée de l'inflation. A l'aide de simulations, nous montrons qu'une grande partie de l'écart obtenu entre les deux estimations MMG et MV peut s'expliquer par une erreur de spécification de type "dynamique omise". Les biais de petit échantillon, parfois invoqués, ne fournissent pas une explication satisfaisante.

Il convient bien sûr de souligner que l'utilisation d'une spécification de type "hybride" n'est pas limitée au cas de la courbe de Phillips ici étudié, et apparaît également dans un grand nombre d'applications macroéconomiques comme des équations de consommation, de formation des stocks, d'investissement, etc. Tester la pertinence des résultats obtenus ici pour ces différents domaines d'application constitue une voie d'approfondissement possible du présent travail.

Non-technical summary:

Many macroeconomic models involve estimating a "hybrid" equation, in which the variable of interest depends on its expected future value and its lagged value. Such a specification has been referred to as hybrid, because it nests a forward-looking specification often derived from a Euler condition as well as the backward-looking autoregressive distributed-lag specification. A prominent example is the hybrid Phillips Curve, in which the inflation rate depends on its own lead and lag and on real marginal cost. This model has been proposed by Fuhrer and Moore (1995), Galí and Gertler (1999) and Christiano, Eichenbaum, and Evans (2001), in order to introduce some inflation persistence in the purely forward-looking model of Taylor (1980), Rotemberg (1982), and Calvo (1983).

Estimation of such an equation under the rational-expectation assumption typically involves either the Generalized-Method-of-Moment (GMM) or the Maximum-Likelihood (ML) approach. GMM and ML are alternative procedures to cope with the unobserved forward-looking component of the hybrid model. On one hand, GMM expresses the expected variable as a function of an instrument set, without referring to the structure of the process driving the forcing variable. On the other hand, ML produces model-consistent forecasts in taking into account the structure of the equation for the forcing variable. While the two approaches are asymptotically equivalent, a recurrent finding is that empirical estimates of the hybrid model produce contrasting results, suggesting that the estimation method plays a role in the conflict. For instance, studies based on the ML approach tend to obtain that the forward-looking component in US inflation is essentially unimportant. In contrast, studies using the GMM approach generally report that the forward-looking component is dominant.

The purpose of the present paper is to rationalize the discrepancy between empirical estimates obtained using the ML and GMM approaches, with a focus on the forward-looking

parameter. We first point that, unless a large number of irrelevant instruments is used, finite-sample biases are not likely to fill the gap between parameter estimates typically obtained in empirical applications. We then show that two natural mis-specifications (measurement error in the forcing variable and omitted dynamics) can produce large discrepancies. These results are established analytically in a stylized representative framework. Interestingly, in most cases, the probability limits of the GMM and ML estimators of the degree of forward-lookingness are biased *in opposite directions* with respect to the true value of the parameter. Using Monte-Carlo simulations, we illustrate that the discrepancy of estimators carries on to more complex models, which cannot be solved analytically. Our results shed some light on the long-lasting empirical debate over the importance of the forward-looking component in the hybrid Phillips curve. Our evidence suggests that the conflict between estimates reported in the empirical literature may be rationalized by an omitted dynamics.

Results in the present paper point to one critical source of the discrepancy in estimators of a hybrid equation: that a relevant forcing variable is omitted from the estimated equation but included in the GMM instrument set. Such an instance, rather likely if a large number of instruments is used, will cause the lead of the dependent variable to capture the effect of the omitted variable, and its parameter to be over-estimated. In the type of set-up analyzed here, mis-specification of the equation of interest is typically found to be more harmful to the GMM estimator than to the ML estimator. This finding to some extent balances the well-known fact that in rational-expectation models, ML may, unlike GMM, suffer from mis-specification of the auxiliary model.

In the empirical part, we use our theoretical results to investigate the contrasting estimates of the hybrid Phillips curve obtained on US data. We actually obtain a gap between GMM and ML estimates of the forward-looking component. Using Monte-Carlo simulations, we show that a large part of the discrepancy between the two estimates can be explained by a mis-specification such as an omitted dynamics. Finite-sample biases, in contrast, are not able to fill the gap between these estimates.

It is worth noticing that the hybrid specification is not restricted to the Phillips curve and may be relevant for a number of macroeconomic variables, such as consumption, stocks, or investment. Investigating the relevance of our results to these various fields is left for further research.

1 Introduction

Many macroeconomic models involve estimating a "hybrid" equation of the form:

$$Y_t = \omega_f E_t Y_{t+1} + \omega_b Y_{t-1} + \beta Z_t + \varepsilon_t, \quad (1)$$

where Y_t denotes the dependent variable, Z_t the forcing variable, ε_t the error term, and E_t the expectation conditional on the available information. Such a specification has been referred to as hybrid, because it nests a forward-looking specification often derived from a Euler condition as well as the backward-looking autoregressive distributed-lag specification.

A prominent example is the hybrid Phillips Curve, in which the inflation rate depends on its own lead and lag and on real marginal cost. This model has been proposed by Fuhrer and Moore (1995), Galí and Gertler (1999), and Christiano, Eichenbaum, and Evans (2001), in order to introduce some inflation persistence in the purely forward-looking model of Taylor (1980), Rotemberg (1982), and Calvo (1983). Several interpretations for the presence of a lag in equation (1) have been put forward in the recent literature on the Phillips curve:

For instance, in a model with staggered contracts, such a specification is obtained when agents care about relative real wages over the life of the wage contract (Fuhrer and Moore, 1995), while some firms using a non-rational rule of thumb to set their price may set their price on the basis of past inflation (Galí and Gertler, 1999, or Amato and Laubach, 2003). Kozicki and Tinsley (2002) also suggest that equation (1) may be viewed as the reduced form of rational-expectation models with frictions on price adjustment. Alternative illustrations of equation (1) may be found in the literature on inventories (Fuhrer, Moore, and Schuh, 1995), on investment (Oliner, Rudebusch, and Sichel, 1996), on consumption (Fuhrer, 2000), or more recently on output gap (Fuhrer and Rudebusch, 2002).

Estimation of equation (1) under the rational-expectation assumption typically involves either the Generalized Method of Moment (GMM) or the Maximum-Likelihood (ML) approach, to cope with the unobserved forward-looking component of the hybrid model.[1] On one hand, GMM expresses the expected variable $E_t Y_{t+1}$ as a function of an instrument set, without referring to the structure of the process driving the forcing variable. On the other hand, ML produces model-consistent forecasts of Y_{t+1} in taking into account the structure of the equation for Z_t. While the two approaches are asymptotically equivalent, a recurrent finding is that empirical estimates of the hybrid model produce contrasting results, suggesting that the estimation method plays a role in the conflict. For instance, Fuhrer (1997), using the ML approach, finds the forward-looking component in US inflation to be essentially unimportant. In contrast, Galí and Gertler (1999), using the GMM approach, report that the forward-looking component is dominant.[2] Similarly, Fuhrer and Rudebusch (2002) estimate a hybrid I-S curve

[1] The present paper does not focus on the comparison of GMM and ML as estimation methods per se, but rather as alternative procedures to perform a projection of a rational-expectation term onto an information set. In particular, we do not put emphasis on issues such as choosing the precise shape of the likelihood function or the optimal GMM weighting matrix. In particular, in the case we consider analytically, GMM reduces to two-stage least-square estimation.

[2] There are some exceptions to this broad picture. Kurmann (2002) obtains a dominant forward-looking component, while using an ML estimation approach. In contrast, Ma (2002) obtains GMM estimates in which lagged inflation is almost as important as expected inflation.

using both estimation procedures, and find that the GMM estimate of the forward-looking parameter is systematically larger than the ML estimate.

The purpose of the present paper is to rationalize the discrepancy between empirical estimates obtained using the ML and GMM approaches, with a focus on the forward-looking parameter ω_f. This parameter is indeed crucial in many applications. For instance, in the new Phillips curve context, the value of this parameter has dramatic implications for policy purposes, since it directly affects the effectiveness of monetary policy (see Fuhrer, 1997). We first point that, unless a large number of irrelevant instruments is used, finite-sample biases are unlikely to fill the gap between parameter estimates typically obtained in empirical applications. We then show that two natural mis-specifications (measurement error in the forcing variable and omitted dynamics) can produce large discrepancies. These results are established analytically in a stylized representative framework. Interestingly, in most cases, the probability limits of the GMM and ML estimators of the degree of forward-lookingness are biased *in opposite directions* with respect to the true value of the parameter. Using Monte-Carlo simulations, we illustrate that the discrepancy of estimators carries on to more realistic models, which cannot be solved analytically. Our results shed some light on the long-lasting empirical debate over the importance of the forward-looking component in the hybrid Phillips curve. Our evidence suggests that the conflict between estimates reported in the empirical literature may be rationalized, to a great extent, by an omitted dynamics.

The issue we are concerned with has been tackled by several recent papers. On one hand, we build on papers which analyze GMM estimators under mis-specification in the context of the new Phillips curve. First, Rudd and Whelan (2001) consider the case where a variable is missing in the estimated regression, but included in the instrument set. Second, Mavroeidis (2001) discusses identification in the new Phillips curve, and investigates mis-specification based on omitted dynamics. On the other hand, Lindé (2001) compares the finite-sample performance of GMM and ML estimates of the hybrid Phillips curve using Monte-Carlo simulations and investigates the consequences of a measurement error in the output gap. In the context of a hybrid Euler equation for output, Fuhrer and Rudebusch (2002) explore the extent of the finite-sample bias of GMM and ML estimates using simulations as well. The present paper extends the previous literature by providing analytical results related to both GMM and ML estimators under mis-specification.

The remainder of the paper is organized as follows. In Section 2, we describe the stylized DGP used in our analysis, and describe the GMM and ML estimators. We also investigate the size of the finite-sample bias of those estimators. Section 3 explores how measurement error in the forcing variable and omitted dynamics affect the estimator bias. Several analytical results concerning the ranking of estimators are proposed. In Section 4, we consider a more general model with some feedback from the dependent variable towards the forcing variable. This case is investigated using Monte-Carlo experiments, since the model cannot be solved analytically. In Section 5, we illustrate that the contrasting findings obtained in the empirical literature on the new Phillips curve can be rationalized using the results obtained in this paper. Section 6 provides concluding remarks.

2 A stylized DGP with a single lag and lead hybrid equation

2.1 The DGP

In this section, we begin with a description of our baseline DGP and of the estimators that will be used in the remaining of the paper. The stylized DGP includes the hybrid equation, in which both a lag and an expected lead of the dependent variable are introduced, as well as an AR(1) forcing variable:

$$Y_t = \omega_f E_t Y_{t+1} + (1-\omega_f) Y_{t-1} + \beta Z_t + \varepsilon_t \quad (2)$$

$$Z_t = \rho Z_{t-1} + u_t. \quad (3)$$

Typically, in the hybrid Phillips curve, Y_t represents inflation and Z_t the (log) real marginal cost. We assume the data to be centered. For convenience, error terms ε_t and u_t are assumed to be contemporaneously and serially uncorrelated white noises, with $\sigma_\varepsilon^2 = E\left(\varepsilon_t^2\right)$ and $\sigma_u^2 = E\left(u_t^2\right)$. Structural parameters are therefore $\xi = \left\{\omega_f, \beta, \rho, \sigma_\varepsilon^2, \sigma_u^2\right\}$, with $\left\{\omega_f, \beta, \sigma_\varepsilon^2\right\}$ the parameters of interest, and $\left\{\rho, \sigma_u^2\right\}$ the nuisance parameters.[3]

In order to obtain analytical solutions, we assume, for the moment, that the dynamics of the forcing variable is given by an AR(1) process, so that Z_t is strongly exogenous with respect to the parameters of interest. In addition, for the parameters to be identified, we also impose that the sum the forward-looking and backward-looking parameters sum to one, so that $\omega_f + \omega_b = 1$. Such an assumption is implied by the theoretical derivations of the hybrid Phillips curve in Fuhrer and Moore (1995) or Christiano, Eichenbaum, and Evans (2001). Moreover, the underlying structural model proposed by Galí and Gertler (1999) implies that $\omega_f + \omega_b$ is very close to, but strictly less than, one.

The properties of the model can be derived from the characteristic polynomial given by $\left(1 - \omega_f L^{-1} - (1-\omega_f) L\right)$. The two roots are $\varphi_1 = (1-\omega_f)/\omega_f$ and $\varphi_2 = 1$. According to the conditions for existence and uniqueness of solution to rational-expectation models, established by Blanchard and Kahn (1980), two situations can be encountered in the case $\omega_f + \omega_b = 1$. When $\omega_f \leq 0.5$ (i.e., $\varphi_1 \geq 1$), the solution is unique, but the process Y_t is non-stationary. When $\omega_f > 0.5$ (i.e., $\varphi_1 < 1$), existence of a stationary solution is guaranteed, but the solution is not unique. Stationarity of the model also requires the forcing variable to be stationary, which implies $|\rho| < 1$.

Since our purpose is to compare the properties of estimators in the context of a hybrid equation, we are primarily interested in stationary models and, therefore, we assume, for the moment, that $\omega_f > 0.5$. Note that, when more general dynamics of the forcing variable are considered, a stationary process can be attained for a wider range of values for ω_f. This issue is addressed in details in Section 4, in which the forcing variable is allowed to depend on Y_{t-1}.[4] Note also that, since in many models ω_f can be viewed as the fraction of forward-looking agents, economic interpretation suggests $\omega_f \leq 1$ (i.e., $\varphi_1 \geq 0$), although this assumption is not necessary from a statistical viewpoint for the process Y_t to be stationary. Our maintained

[3] These parameters are considered as structural here, but they are usually defined as functions of "deep" parameters which reflect constraints and preferences of agents.

[4] In the empirical part of the paper, we will also relax our assumption and allow $\omega_f + \omega_b$ to be smaller than one.

assumptions for the model (2)–(3) are thus $0.5 < \omega_f \leq 1$ (or, equivalently, $0 \leq \varphi_1 < 1$) and $|\rho| < 1$.

Under stationarity, the reduced-form, fundamental solution of the DGP is given by

$$Y_t = \varphi_1 Y_{t-1} + \theta Z_t + \tilde{\varepsilon}_t \tag{4}$$

where $\theta = \beta / (\omega_f (1-\rho))$, and $\tilde{\varepsilon}_t = \varepsilon_t / \omega_f$. We define $\tilde{\sigma}_\varepsilon^2 = E\left(\tilde{\varepsilon}_t^2\right) = \sigma_\varepsilon^2 / \omega_f^2$.

Before discussing the way estimators are constructed, we need to address the important issue of non-uniqueness of the solution which occurs when $\omega_f > 0.5$. The general form representation is given by solving equation (2) for Y_{t+1}, which yields

$$Y_t = \frac{1}{\omega_f} Y_{t-1} - \frac{1-\omega_f}{\omega_f} Y_{t-2} - \frac{\beta}{\omega_f} Z_{t-1} - \frac{1}{\omega_f} \varepsilon_{t-1} + \zeta_t \tag{5}$$

where $\zeta_t = Y_t - E_{t-1} Y_t$ is the agents' forecast error. Under rational expectations, the process ζ_t is a martingale difference sequence, such that $E_{t-1}\zeta_t = 0$. The general solution (5) always satisfies equation (2). However, it reduces to the forward-looking solution (4) only when $\zeta_t = \tilde{\varepsilon}_t + \theta u_t$.

The generic solution is then given by

$$Y_t = \varphi_1 Y_{t-1} + \theta Z_t + \tilde{\varepsilon}_t + \xi_t \tag{6}$$

where ξ_t satisfies[5]

$$\xi_t = \xi_{t-1} - \tilde{\varepsilon}_t - \theta u_t + \zeta_t. \tag{7}$$

An important consequence of this result is that, among the infinite set of solutions to the problem (2)–(3), only the sunspot-free (fundamental) solution (4) defines a stationary process Y_t. This result comes from that equation (7) defines a non-stationary process ξ_t.

In the remainder, we will focus on the stationary solution for the process Y_t, given by equation (4). In the new Phillips curve context, as in other applications, it is economically reasonable to rule out non-stationary bubbles. Note, in addition, that such a non-stationary representation for a hybrid model is very unlikely to occur in a more general dynamic equilibrium model with rational expectations.[6]

[5] This result is obtained by observing that the general form representation (5) can be rewritten, using equation (7)

$$\begin{aligned} Y_t &= \frac{1}{\omega_f} Y_{t-1} - \frac{1-\omega_f}{\omega_f} Y_{t-2} - \frac{\beta}{\omega_f} Z_{t-1} - \frac{1}{\omega_f} \varepsilon_{t-1} + \left(\xi_t - \xi_{t-1} + \tilde{\varepsilon}_t + \theta u_t\right) \\ (1-L)(1-\varphi_1 L) Y_t &= \frac{\beta}{\omega_f (1-\rho)} (Z_t - Z_{t-1}) + (\tilde{\varepsilon}_t - \tilde{\varepsilon}_{t-1}) + \left(\xi_t - \xi_{t-1}\right) \\ Y_t &= \varphi_1 Y_{t-1} + \frac{\beta}{\omega_f (1-\rho)} Z_t + \tilde{\varepsilon}_t + \xi_t, \end{aligned}$$

which is equation (6).

[6] For instance, in the model proposed by Christiano, Eichenbaum, and Evans (2001), the hybrid Phillips curve is designed as our equation (2), with $\omega_f = \delta / (1+\delta)$, $\omega_b = 1/(1+\delta)$, and $\omega_f + \omega_b = 1$. In addition, δ is the discount factor, so that ω_f is lower than 0.5. Therefore, in a set-up like equations (2)–(3), even with a stationary forcing variable, the Blanchard-Kahn conditions indicate that the inflation process would be non-stationary. In the complete model, however, the process for the forcing variable allows for feedback from inflation, so that the inflation process is readily stationary.

An alternative way to single out the rational-expectation solution (4) is to resort to the minimum state variable approach developed by McCallum (1983), who argues that agents will include in their forecasting rules a minimal set of state variables.[7] A definite advantage of this procedure is that it does not rely upon any assumption or condition concerning the dynamic stability of the system. In our context, the minimum state variable approach precludes Y_{t-2} from the reduced-form solution (5).

2.2 Estimators

Since one of the regressors (the expected term) is correlated to the error term, OLS estimation of the equation

$$Y_t = \omega_f Y_{t+1} + (1 - \omega_f) Y_{t-1} + \beta Z_t + \varepsilon_t',$$

where $\varepsilon_t' = \varepsilon_t - \omega_f (Y_{t+1} - E_t Y_{t+1})$, yields inconsistent estimators. Two alternative estimation procedures can then be considered to cope with this problem, GMM and ML.

2.2.1 The GMM estimator

The GMM approach reduces to the two-stage least-square estimation in this framework. It consists in regressing Y_{t+1} on instruments which are uncorrelated with the error term ε_t' but correlated with the endogenous regressor (Y_{t+1}). Since two parameters (φ_1 and θ or, equivalently, ω_f and β) have to be estimated, at least two instruments are needed to achieve identification. Assuming the econometrician knows the true specification (2), but does not want to specify the dynamics of the forcing variable, the optimal GMM estimator is obtained using as instrument set $\{Y_{t-1}, Z_t\}$. This estimator relies on the following moment conditions:

$$\begin{aligned} E\left[Y_{t-1} \cdot (Y_t - \omega_f Y_{t+1} - (1 - \omega_f) Y_{t-1} - \beta Z_t)\right] &= 0 \\ E\left[Z_t \cdot (Y_t - \omega_f Y_{t+1} - (1 - \omega_f) Y_{t-1} - \beta Z_t)\right] &= 0. \end{aligned}$$

Since the model is just-identified, the probability limits (Plims) of the estimator are directly obtained by solving these moment conditions. As Z_t is in the instrument set, solving moment conditions is equivalent to the following two-step problem: First, Y_{t+1} is regressed on Y_{t-1} and Z_t giving the expectation $\widehat{Y}_{t+1}$ conditional on the instrument set. Second, $(Y_t - Y_{t-1})$ is regressed on $\left(\widehat{Y}_{t+1} - Y_{t-1}\right)$ and Z_t, yielding consistent parameter estimators.

2.2.2 The ML estimator

The second approach is the ML procedure, which relies on using equation (3) to solve equation (2) iteratively forward. Estimating the reduced-form equation (4) together with equation (3) and imposing cross-equation restrictions allow one to recover the structural parameters. Parameters φ_1 and θ in equation (4) and ρ in equation (3) can be estimated by OLS directly. Then, estimators of ω_f and β are deduced from the relations $\omega_f = 1/(1 + \varphi_1)$ and $\beta = \theta \omega_f (1 - \rho)$. Since estimation is performed after the hybrid model has been solved iteratively forward, ML estimators are obtained under the assumption that forecasts are fully model-consistent. The crucial difference between GMM and ML approaches is that ML imposes some

[7] This argument has been reinforced by the literature on learning in rational-expectation models, which shows that the minimum state variable solution is the only stable solution (Evans and Honkapohja, 1998).

constraints upon the way Y_{t+1} is projected onto the state variables, through the dynamics of Z_t used to solve the model. In contrast, GMM does not impose any constraint of this type on the first-stage regression.

Since innovations are assumed to be uncorrelated, this two-step approach is equivalent to the full-information ML. When the model is more realistic (for instance, when a feedback from Y_{t-1} to Z_t is allowed), the model generally cannot be solved analytically (see Section 4).

The GMM and ML estimators presented above have the same Plim when the estimated model is correctly specified. Yet, in many empirical applications of the hybrid model, the gap between GMM and ML estimates has been found to be very large. Two reasons are likely to explain such a discrepancy between GMM and ML estimators: (1) There may be differences in the finite-sample properties of the estimators. (2) The estimated model may be mis-specified, yielding inconsistency of GMM as well as ML estimators. The remaining of the paper considers these two explanations in turn.

2.3 Finite-sample biases

This section investigates the finite-sample properties of the GMM and ML estimators assuming that the model is correctly specified. As in earlier studies, we rely on Monte-Carlo simulations to evaluate the finite-sample distribution of the estimators.

An abundant literature has studied the finite-sample properties of the GMM estimator, in very different contexts (see Fuhrer, Moore, and Schuh, 1995, or the 1996 special issue of the JBES). These papers provided evidence that the GMM estimator may be strongly biased and widely dispersed in finite sample. The size of this bias is related to weak instrument relevance (i.e., weak correlation between instruments and endogenous regressors) or to instrument redundancy. This issue has been addressed, among others, by Nelson and Startz (1990), Hall, Rudebusch, and Wilcox (1996), or Staiger and Stock (1997). The ML estimator may suffer from finite-sample bias as well. Few studies have focused on the finite-sample properties of the ML estimator in rational-expectation models (see Fuhrer, Moore, and Schuh, 1995, Lindé, 2001, Fuhrer and Rudebusch, 2002, Jondeau, Le Bihan, and Gallès, 2003). The ML finite-sample bias is generally considered to be negligible in this context.[8]

When the GMM approach is implemented, we have to select the instrument set. The optimal instrument set in our set-up is $W_t = \{Y_{t-1}, Z_t\}$. This is our baseline case in the following experiments. We also explore the case where lags of W_t are introduced in the instrument set, although they are actually redundant. Thus, we use instrument sets which include W_t, ..., W_{t-L}, with $L = 0$ and 7. It is worth emphasizing that it is a common practice in the empirical GMM literature to include several lags, without necessarily checking their relevance. A dramatic drawback of this practice is the aforementioned weak instrument relevance or instrument redundancy. It can be shown (Nelson and Startz, 1990) that the GMM estimator is biased in direction of the OLS estimator (see also Staiger and Stock, 1997). We thus have also considered the Plim of the (mis-specified) OLS estimator, which provides an indication of the effect of weak instrument relevance on the finite-sample bias of GMM

[8] The ML estimator is not necessarily immune from finite-sample bias, however. In particular, in a partial-adjustment context, the autoregressive parameter is biased downward, even when the model is correctly specified (Sawa, 1978). Yet, this bias is rarely emphasized in Monte-Carlo experiments.

estimates. Using the cross moments implied by DGP (2)–(3) (see Appendix 1), the Plims of OLS estimators are found to be:

$$\omega_{OLS} = \frac{1}{2}\left(\frac{\tilde{\sigma}_{\varepsilon}^{2}+\Lambda^{2}\left(1-\varphi_{1}\right)}{\tilde{\sigma}_{\varepsilon}^{2}+\Lambda^{2}\left[\frac{1}{2}\left(1-\varphi_{1}^{2}\right)+\left(1-\varphi_{1}\rho\right)^{2}\right]}\right)$$

$$\beta_{OLS} = \left(\frac{\beta}{a}\right)\frac{1+\varphi_{1}}{1-\rho}\left(\frac{\tilde{\sigma}_{\varepsilon}^{2}+\Lambda^{2}\left[\frac{2(1-\rho)-\varphi_{1}\left(1-\rho^{2}\right)}{1-\varphi_{1}\rho}\right]}{\tilde{\sigma}_{\varepsilon}^{2}+\Lambda^{2}\left[\frac{1}{2}\left(1-\varphi_{1}^{2}\right)+\left(1-\varphi_{1}\rho\right)^{2}\right]}\right),$$

where $\Lambda = \theta\sigma_u/\left(1-\varphi_1\rho\right)$. For a wide range of structural parameters, the OLS estimator of ω_f is biased towards 0.5, although the Plim is not exactly 0.5. More precisely, the expression above indicates that $0 \leq \omega_{OLS} \leq 0.5$. Such a result has been obtained numerically in several Monte-Carlo studies, but not yet analytically. See, for instance, Lindé (2001), Mavroeidis (2001), and Fuhrer and Rudebusch (2002).

2.3.1 Experiment design

Essentially, four structural parameters are likely to affect the finite-sample bias: The forward-looking parameter ω_f, the serial correlation of the forcing variable ρ, the parameter of the forcing variable β, and the (square root of the) variance ratio $\sigma_u/\sigma_\varepsilon$. We consider the following parameter sets in Monte-Carlo experiments: $\omega_f = \{0.55; 0.75; 0.95\}$, $\rho = \{0.1; 0.5; 0.9\}$, $\beta = \{0.1; 1\}$, and $\sigma_u/\sigma_\varepsilon = \{0.5; 1; 2\}$. Since we found that altering the variance ratio within this range does not affect finite-sample biases significantly, only the case $\sigma_\varepsilon = \sigma_u = 1$ is reported.

The Monte-Carlo experiment is performed as follows. For each parameter set, we simulate 2000 samples of size $T = 100$. For each simulated sample, a sequence of $T + 100$ random innovations are drawn from the Gaussian distribution $N(0, \Sigma)$ with no serial correlation ($\Sigma = diag(\sigma_\varepsilon^2, \sigma_u^2)$), and the first 100 entries are discarded to reduce the effect of initial conditions on the solution path.

2.3.2 Results

Table 1 reports the median and the median of absolute deviations (MAD) of the empirical distribution.[9] It also displays the Plim of the OLS estimator, towards which the Plim of GMM estimator is biased, under weak instrument relevance or instrument redundancy. The main results are as follows.

First, the ML estimator of ω_f is essentially unbiased, with a low standard deviation. The estimator of β is unbiased for small values of ρ, while there is a slight positive bias when ρ is large.[10]

[9] One justification of the use of the median and MAD is that, in the just-identified case, the distribution of the GMM estimator has no finite moments (Kinal, 1980). In addition, previous research on the finite-sample properties of GMM has found the distribution of this estimator to be sometimes asymmetric and fat tailed (Hansen, Heaton, and Yaron, 1996).

[10] For $\rho = 0.9$, the median estimator of β ranges between 0.107 and 0.112, whatever the true parameter ω_f. This bias is related to the downward bias in the autoregressive parameter established analytically by Sawa (1978). Since $\beta = \theta\omega_f(1-\rho)$, the negative bias in ρ translates in a positive bias in β. Note that, since ω_f does not depend on ρ, we do not observe such a bias for ω_f.

Second, when the instrument set is correctly chosen ($L = 0$), the GMM estimator of ω_f is not significantly biased. However, the standard deviation of the estimator (computed as the standard deviation of the distribution) is much larger than the standard deviation of the ML estimator.

Last, when the instrument set includes redundant lags ($L = 7$), the bias in the GMM estimator of ω_f is significantly negative. The higher the true value of ω_f, the larger the absolute bias in the estimator. For $\omega_f = 0.95$, the absolute finite-sample bias is as high as 0.1. In fact, the median estimate lies between the true value of the parameter and the Plim of the OLS, which is found to be close to 0.5 for a wide range of structural parameters. For large ρ, the bias in β is as high as 30% of the true value.

Result 1: *When the model is correctly specified, the ML estimator does not display significant finite-sample bias. The GMM estimator with optimal instruments is not significantly biased, while the GMM estimator with an excessive number of instruments is significantly biased towards the OLS estimator (close to 0.5).*

This result confirms previous evidence obtained for instance by Mavroeidis (2001) and Lindé (2001). Fuhrer and Rudebusch (2002) also report simulation results in which the GMM estimator of ω_f is biased towards 0.5. It is worth noting, however, that the finite-sample bias in ω_f is not likely to reconcile the conflicting empirical evidence observed for the hybrid Phillips curve. Indeed, the GMM estimator is systematically biased towards (around) 0.5 in our Monte-Carlo set-up, while estimates reported, for instance, by Galí and Gertler (1999) suggest a bias, if any, towards one. In addition, the ML estimator does not display any substantial finite-sample bias. Therefore, if ML estimates were to be close to the true values of the DGP parameters, the GMM estimate of ω_f would be found to be lower than the ML estimate, so that finite-sample bias would affect estimators in a direction opposite to that suggested by empirical results in the existing literature.

3 Asymptotic bias in a mis-specified model

This section investigates the consequences of two types of mis-specification, measurement error and omitted dynamics. Both mis-specifications appear to be plausible in many applications of the hybrid model.

First, estimated new Phillips curves may suffer from measurement error. Theoretical derivations suggest that the relevant forcing variable is the real marginal cost. While this variable may be measured in a variety of fashions, empirical estimates typically involve either the output gap or real unit labor cost (ULC) (Roberts, 2001, and Neiss and Nelson, 2002). The approximation of real marginal cost by real ULC readily arises when firms produce using a Cobb-Douglas technology with constant return to scale. Under some additional assumptions about the labor supply process (Rotemberg and Woodford, 1997), the output gap is linearly related to the real marginal cost. However, the measure of these proxies is also an open issue. As detailed by Rotemberg and Woodford (1999), the real ULC measure has to be corrected in various ways to approximate the real marginal cost under arguably more realistic technology assumptions (such as a CES technology or overhead labor). In addition, conventional measures of output gap are likely to be ridden with error. The standard approach typically involves a deterministic trend, which may fail to capture variations in the natural rate of output due, e.g., to supply shocks. See Neiss and Nelson (2002) for a discussion and an alternative measure of potential output based on a dynamic stochastic general equilibrium model.

Omitted dynamics is also a plausible mis-specification. Hybrid Phillips curves in which additional lags and leads of inflation are incorporated were first suggested by Fuhrer and Moore (1995) and Fuhrer (1997). Their model assumes that agents set nominal contract prices so that the current real contract index depends on the real contract index expected to prevail over the life of the contract. Therefore, in cases where prices are set for several quarters at a time, agents are concerned by lags and leads of inflation symmetrically. Coenen and Wieland (2000) also adopt such an approach, while Sbordone (2002) as well as Guerrieri (2001) consider a model with optimizing firms, in which price commitments last for a fixed length of several periods. Galí, Gertler, and López-Salido (2001) propose a model which extends the model developed in Galí and Gertler (1999): In contrast to the initial rule of thumb which was assumed to depend on the last period's inflation rate only, firms which adopt a non-rational behavior consider an average of several lags. Kozicki and Tinsley (2002) have recently discussed the empirical performances of alternative potential sources of lag dynamics in inflation (namely, non-rational behavior, staggered contracting, frictions on price adjustment and shifts in the long-run anchor of agent expectations). They obtain that additional lags (and leads) explain the historical behavior of inflation in the US and Canada better than a purely forward-looking model or a hybrid model with a single lag. In addition, models derived from assumptions about staggered contracts or frictions on price adjustment provide a better fit of the data than hybrid models with non-rational behavior. Rationalization for additional lags in hybrid equations of the type (1) can also be found in other contexts. For instance, Otrok, Ravikumar, and Whiteman (2002) have explored general forms of habit formation in the consumption function, which can lead to a reduced form with several lags of consumption.

3.1 Mis-specification of type I: Measurement error

3.1.1 Analytical results

In order to illustrate the case of measurement error, we adopt the following DGP:

$$\begin{aligned} Y_t &= \omega_f E_t Y_{t+1} + (1-\omega_f) Y_{t-1} + \beta Z_t + \varepsilon_t \\ Z_t &= \rho Z_{t-1} + u_t \\ X_t &= a Z_t + e_t \end{aligned} \tag{8}$$

where X_t is the proxy of the forcing variable used by the econometrician (e.g., the output gap or the real ULC in the context of the hybrid Phillips curve, while the relevant forcing variable Z_t is the real marginal cost). The measurement error, e_t, is assumed to be contemporaneously and serially uncorrelated with ε_t and u_t, with $\sigma_e^2 = E\left(e_t^2\right)$. We also define $\tau = a^2\sigma_Z^2 / \left(a^2\sigma_Z^2 + \sigma_e^2\right)$, the fraction of the variance of X_t explained by Z_t. Parameter τ can be viewed as a measure of the quality of the proxy X_t. When τ gets closer to one, the quality of the proxy improves. While parameter a may be positive as well as negative, it is typically set equal to one in the following. DGP parameters are $\xi = \left\{\omega_f, \beta, \rho, a, \sigma_\varepsilon^2, \sigma_u^2, \sigma_e^2\right\}$. The reduced form of the DGP is given by equation (4) with the same stationarity conditions on model parameters.

Since the econometrician is assumed to erroneously select X_t as the forcing variable, the following mis-specified model is estimated:[11]

$$Y_t = \alpha_f E_t Y_{t+1} + (1-\alpha_f) Y_{t-1} + bX_t + v_t \tag{9}$$
$$X_t = \psi X_{t-1} + w_t. \tag{10}$$

The estimated degree of forward-lookingness α_f is presumably a biased estimator of ω_f. Note that there is no mis-specification in the limiting case where $\sigma_e^2 = 0$, i.e. $\tau = 1$, so that X_t is actually the true forcing variable.

The reduced form of the postulated system (9)–(10) is given by

$$Y_t = \varphi Y_{t-1} + \mu X_t + \tilde{v}_t, \tag{11}$$

where $\varphi = (1-\alpha_f)/\alpha_f$, $\mu = b/(\alpha_f(1-\psi))$, and $\tilde{v}_t = v_t/\alpha_f$. As in Section 2, we impose the following conditions: $0.5 < \alpha_f \leq 1$ (or $0 \leq \varphi < 1$) and $|\psi| < 1$.

Since two parameters (α_f and b) have to be estimated in the mis-specified model, at least two instruments are needed to achieve identification. We consider two cases of particular interest. The first estimator (GMM1) is based on the instrument set $\{Y_{t-1}, X_t\}$, while the second estimator (GMM2) resorts to the instrument set $\{Y_{t-1}, X_t, Z_t\}$. In the latter case, the instrument set includes the actual forcing variable. Both cases are likely to occur in empirical applications. For instance, the first estimator may correspond to the case of the hybrid I-S curve, as studied recently by Fuhrer and Rudebusch (2002). Instruments are lagged output gap, inflation rate, and interest rate. Mis-specification may occur in this context because of an appropriate definition of the real interest rate. In contrast, in the case of the hybrid Phillips curve, the theoretically relevant forcing variable is the real marginal cost, while estimations are performed using, alternatively, the output gap and the real ULC. Most studies (in the following of Galí and Gertler, 1999) therefore include both the real ULC and output gap in the instrument set. We may expect the real marginal cost to be well proxied by a linear combination of the two variables.

GMM1. Estimator GMM1 relies on the following moment conditions:

$$E\left[Y_{t-1} \cdot \left(Y_t - \alpha_f Y_{t+1} - (1-\alpha_f) Y_{t-1} - bX_t\right)\right] = 0 \tag{12}$$
$$E\left[X_t \cdot \left(Y_t - \alpha_f Y_{t+1} - (1-\alpha_f) Y_{t-1} - bX_t\right)\right] = 0. \tag{13}$$

Parameter estimates are obtained by solving the empirical counterparts of these moment conditions. The Plim of the estimator of α_f is given by

$$\alpha_{GMM1} = \frac{E\left(Y_t^2\right) - E\left(Y_t Y_{t-1}\right) + \left(E\left(X_t Y_t\right)/E\left(X_t^2\right)\right)\left(E\left(X_t Y_t\right) - E\left(X_t Y_{t-1}\right)\right)}{E\left(Y_t^2\right) - E\left(Y_t Y_{t-2}\right) + \left(E\left(X_t Y_t\right)/E\left(X_t^2\right)\right)\left(E\left(X_t Y_{t-1}\right) - E\left(X_{t-1} Y_t\right)\right)},$$

[11] Combining equations (3) and (8) should yield an ARMA process for X_t, because the measurement error introduces a first-order serial correlation in the error term of X_t. In the case $a = 1$, the process for X_t can be written as $X_t = \rho X_{t-1} + w_t - cw_{t-1}$, with $c = \left(\sigma_w^2 \pm \sqrt{\sigma_w^4 - 4\rho^2\sigma_e^4}\right) / \left(2\rho\sigma_e^2\right)$ and $\sigma_w^2 = \rho\sigma_e^2/c$. Such an approach is not followed, however, because it would not be consistent with the standard approach of a low-order VAR-type model. Estimating equation (10), thus omitting the MA component, yields a Plim of the autoregressive parameter ψ equal to $\rho\tau$.

while the Plim of the estimator of b, denoted b_{GMM1}, is obtained by replacing α_f by α_{GMM1} in the expression (13). Although the moment conditions are written using the mis-specified model (9), the moments are actually computed using the true DGP.

GMM2. Since estimator GMM2 is over-identified, it proves convenient to view it as a two-step estimator.[12] First, we regress Y_{t+1} on the instrument set to build the fitted value. For simplicity of the exposition, we define $\widehat{Y}_{t+1}$ as the expectation of Y_{t+1} conditional on the information set. Since the true forcing variable Z_t is in the instrument set, we obtain, using equations (3) and (4), that $\widehat{Y}_{t+1} = \varphi_1^2 Y_{t-1} + \theta(\varphi_1 + \rho) Z_t$. Then, the second-stage regression has the form

$$Y_t - Y_{t-1} = \alpha_f \left(\widehat{Y}_{t+1} - Y_{t-1}\right) + bX_t + v_t' \tag{14}$$

which can be estimated by OLS. Plims of estimators of α_f and b are thus obtained by solving the two following moment conditions:

$$E\left[\left(\widehat{Y}_{t+1} - Y_{t-1}\right) \cdot \left(Y_t - \alpha_f \widehat{Y}_{t+1} - (1-\alpha_f) Y_{t-1} - bX_t\right)\right] = 0 \tag{15}$$

$$E\left[X_t \cdot \left(Y_t - \alpha_f \widehat{Y}_{t+1} - (1-\alpha_f) Y_{t-1} - bX_t\right)\right] = 0. \tag{16}$$

ML. The ML estimator is obtained by estimating by OLS the reduced form of the model, given by equations (10) and (11). Plims of the estimators of φ and μ are directly given by

$$\varphi_{ML} = \frac{E\left(X_t^2\right) E\left(Y_t Y_{t-1}\right) - E\left(X_t Y_{t-1}\right) E\left(X_t Y_t\right)}{E\left(X_t^2\right) E\left(Y_t^2\right) - E\left(X_t Y_{t-1}\right)^2} = \varphi_1 \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{\rho(1-\varphi_1^2)}{\varphi_1(1-\rho^2)}\right]}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{\rho^2(1-\varphi_1^2)}{(1-\rho^2)}\right]} \right)$$

$$\mu_{ML} = \frac{E\left(Y_t^2\right) E\left(Y_t X_t\right) - E\left(X_t Y_{t-1}\right) E\left(Y_t Y_{t-1}\right)}{E\left(X_t^2\right) E\left(Y_t^2\right) - E\left(X_t Y_{t-1}\right)^2} = \frac{\theta\tau}{a} \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{(1-\varphi_1^2)\rho^2}{(1-\rho^2)}\right]} \right)$$

where the last equation indicates that the estimator of μ is a biased estimator of θ.

Then, the Plims of parameters α_f and b are deduced from the relations $\alpha_f = 1/(1+\varphi)$ and $b = \mu\alpha_f(1-\psi)$.

Since the hybrid model is estimated using X_t as forcing variable in place of Z_t, there exists an asymptotic bias, the extent of which is directly related to the correlation between the two variables X_t and Z_t. The following proposition summarizes the Plims of GMM as well as ML estimators described above.

Proposition 1 (Plim of estimators in case of measurement error) *Let us assume that the DGP is given by equations (2)–(3), but that the econometrician estimates the model with X_t in place of Z_t as the forcing variable, corresponding to equations (9)–(10). Then, the three estimators defined above have the following Plims:*

[12] To obtain analytical results, we abstract from the issue of selecting an optimal weighting matrix. In a correctly-specified model, the two-step least-square approach adopted here yields a consistent estimator of the true parameter. As put forward by Hall and Inoue (2003), in case of mis-specification, different weighting matrices may yield different asymptotic biases. We do not consider this issue here, and leave it for further investigation.

- *GMM estimator with instrument set* $\{Y_{t-1}, X_t\}$ *(GMM1):*

$$\alpha_{GMM1} = \omega_f \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 - (1-\tau)\frac{\rho(1+\varphi_1)}{1+\rho}\right]}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 - (1-\tau)\rho\varphi_1\right]} \right)$$

$$b_{GMM1} = \left(\frac{\beta}{a}\tau\right) \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 - (1-\tau)\rho\varphi_1\right]} \right)$$

- *GMM estimator with instrument set* $\{Y_{t-1}, X_t, Z_t\}$ *(GMM2):*

$$\alpha_{GMM2} = \omega_f \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{(1-\rho)\varphi_1}{1-\varphi_1}\right]}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{(1-\rho^2)\varphi_1^2}{(1-\varphi_1^2)}\right]} \right)$$

$$b_{GMM2} = \left(\frac{\beta}{a}\tau\right) \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{(1-\rho^2)\varphi_1^2}{(1-\varphi_1^2)}\right]} \right)$$

- *ML estimator:*

$$\alpha_{ML} = \omega_f \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{(1-\varphi_1^2)\rho^2}{1-\rho^2}\right]}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{(1-\varphi_1)\rho}{1-\rho}\right]} \right)$$

$$b_{ML} = \left(\frac{\beta}{a}\tau\right) \frac{1-\rho\tau}{1-\rho} \left(\frac{\tilde{\sigma}_\varepsilon^2 + \Lambda^2}{\tilde{\sigma}_\varepsilon^2 + \Lambda^2 \left[1 + (1-\tau)\frac{(1-\varphi_1)\rho}{1-\rho}\right]} \right)$$

where $\Lambda = \theta\sigma_u / (1-\varphi_1\rho)$ *and* $\rho\tau$ *is the Plim of the autoregressive parameter of* X_t *implied by equation (10).*

Proof: See the discussion above. Computation of the Plims abundantly resorts to the moments and cross-moments reported in Appendix 1.

Parameter Λ^2 can be interpreted as the fraction of the variance of Y_t explained by Z_t. We verify that, in the case $\tau = 1$, all estimators of α_f are asymptotically unbiased, so that $\alpha_{GMM1} = \alpha_{GMM2} = \alpha_{ML} = \omega_f$. Similarly, all estimators of b are asymptotically unbiased, up to the scale factor a, since $b_{GMM1} = b_{GMM2} = b_{ML} = \beta/a$.

The first term of the Plim of the estimator of α_f is the asymptotic value of α_f when the model is correctly specified, while the second term reflects the mis-specification bias. Assessing the bias in b involves comparing the estimator to β/a, since X_t may be a good proxy for Z_t, yet having a parameter a different from 1. In addition, X_t being only a proxy of Z_t, b may be viewed as an estimator of $(\beta\tau/a)$. Two sources of bias operate for GMM estimators: The first bias comes from τ being less than one, which measures whether X_t is a good proxy for Z_t. The second bias, which comes from the second term between brackets, measures the extent of the mis-specification bias. For the ML estimator of b, a third source of bias exists. It comes from the expression $(1-\rho\tau)/(1-\rho)$, which reflects the mis-measurement in the serial correlation of the forcing variable. This component always affects the Plim of the estimator positively.

We now provide the ranking of the Plims of GMM and ML estimators in the typical case where the serial correlation of the forcing variable (ρ) is positive.

Corollary 1 *When $0 \leq \rho \leq 1$, the following inequalities hold:*

$$\begin{aligned} 1/4 &\leq \alpha_{GMM1} \leq \omega_f \\ 1/2 &\leq \alpha_{ML} \leq \omega_f \leq \alpha_{GMM2} \leq 1 \end{aligned}$$

and when $(\rho - \varphi_1)(1 - \varphi_1 \rho)^2 (1 - \rho)^2 \leq \rho (1 - \varphi_1^2) \left[(1 - \tau) \beta^2 \sigma_u^2 / \sigma_\varepsilon^2 \right]$*:*

$$1/4 \leq \alpha_{GMM1} \leq \alpha_{ML} \leq \omega_f.$$

In addition, the following inequalities hold:

$$b_{GMM2} \leq (\beta \tau / a) \leq b_{GMM1} \leq b_{ML}.$$

These inequalities are readily obtained by comparing terms between brackets in the numerator and denominator of each expression. Bounds for the Plims of estimators of α_f are obtained, under stationarity, by choosing bounds for structural parameters ω_f (0.5 and 1), ρ (0 and 1), and $\tau = \sigma_\varepsilon^2 = 0$.[13]

We observe that, in the case $\rho = 0$, we have $\alpha_{GMM1} = \alpha_{ML} = \omega_f$, (and $b_{GMM1} = b_{ML} = \beta/a$), while $\alpha_{GMM2} > \omega_f$ and $b_{GMM2} < \beta/a$. The absence of bias for GMM1 and ML derives from orthogonality between the forcing variable and Y_{t-1}, since $\rho = 0$ implies $E(Z_t Y_{t-1}) = E(X_t Y_{t-1}) = 0$. As for GMM1, in the general case ($\rho \neq 0$), the upward bias in the coefficient of Y_{t-1} (i.e., $1 - \alpha_{GMM1}$) reflects the information content of Y_{t-1} on the omitted variable Z_t (because $\rho \neq 0$ implies $E(Z_t Y_{t-1}) \neq 0$, see Appendix 1). When $\rho = 0$, the bias vanishes. The bias also vanishes for the ML estimator, because under condition $\rho = 0$ we have $E(Z_t Y_{t-1}) = E(X_t Y_{t-1}) = 0$, so that regressors in the reduced form are orthogonal. Hence, parameter φ is estimated without bias, and so is the structural parameter recovered from parameter $\alpha_f = 1/(1 + \varphi)$. In the case of the GMM2, however, an upward bias in α_f is still present. Indeed, this bias is related to $E(Y_{t+1} Z_t)$ being different from zero (see the discussion below), which generally remains true when $\rho = 0$.

Result 2: *In the case of measurement error of the type described by equations* (2), (3), *and* (8), *when $0 \leq \rho \leq 1$, ML and GMM2 estimators of α_f are always biased asymptotically in opposite directions with respect to the true value of parameter ω_f.*

3.1.2 Evidence and discussion

Table 2 presents Plims of GMM and ML estimators in the measurement-error case, using formulae reported in Proposition 1. We select as structural parameters $\omega_f = \{0.55; 0.75; 0.95\}$, $\rho = \{0.1; 0.5; 0.9\}$, $\beta = \{0.1; 1\}$, $a = 1$, and $\sigma_u / \sigma_\varepsilon = 1$. Last, we choose $\tau = \{0.1; 0.5; 0.9\}$, which represents the extent to which the variable X_t is a good proxy for Z_t. Parameter τ plays presumably a crucial role regarding asymptotic biases.

[13] These bounds are reported in the bottom part of Table 2.

As regards parameter α_f, illustrating Proposition 1, GMM1 and ML estimators are systematically lower than the true value ω_f, while the GMM2 estimator is systematically larger than ω_f. Since the estimators are bounded by 0.5, GMM1 and ML biases are, as expected, very small when ω_f is equal to 0.55 and, conversely, the GMM2 bias is very small when ω_f is equal to 0.95. The ranking of GMM1 and ML depends on the values of model parameters. When ω_f is small, we obtain a larger bias in GMM1 than in ML. In contrast, when ω_f is large, the GMM1 bias is smaller than the ML bias.

As expected, the lower the quality of the proxy, the larger the bias in α_f. An interesting result is that, for low values of τ and ρ, the GMM2 bias in α_f can be extremely large. For instance, for $\tau = \rho = 0.1$ and $\beta = 1$, when the true forward-looking parameter is in fact 0.55, the Plim is as high as 0.84, so that the dependent variable Y_t would be claimed to be an essentially forward-looking process. Conversely, the ML estimator can be severely biased for low value of τ and large value of ρ. For $\tau = 0.1$, $\rho = 0.9$, and $\beta = 1$, when Y_t is in fact essentially forward-looking ($\omega_f = 0.95$), the Plim is 0.53, so that the forward-looking weight would be claimed to be at its lower bound.

Last, concerning parameter b, the reported Plims illustrate the ranking highlighted in Corollary 1, since they are systematically larger for the ML estimator than for the GMM estimator. In addition, they are very close to $\beta\tau$ when ω_f is large (0.75 or 0.95), but they are likely to be very far from the true parameter for small ω_f. It is worth emphasizing that negative Plims for the estimator of b are precluded (except for $a < 0$).

To summarize, under measurement error, GMM estimation can lead to an over-estimation of the degree of forward-lookingness. The bias is potentially large. This finding echoes the result obtained by Rudd and Whelan (2001), although these authors considered omitted variables rather than measurement errors. Note that our set-up is, in some respect, more general since both the DGP and the estimated equation are hybrid models, while Rudd and Whelan considered the case of a purely backward-looking DGP and a purely forward-looking estimated equation. The crucial feature in the two cases is that a positive GMM bias occurs when the relevant forcing variable is not introduced as regressor in the model, but *is included in the instrument set*. The mechanics is that actual future inflation used in the first-stage regression captures the effect of the (omitted) relevant variable. Then, the second-stage regression tends to put an excessive weight on the fitted value $\hat{Y}_{t+1}$. In contrast, the bias in the ML estimator is related to the standard omitted-variable bias: Estimating equation (11) by OLS puts an excessive weight on Y_{t-1}, since Y_{t-1} is, in cases we consider, positively correlated with Z_t. The result that the biases of the GMM and ML estimators are in opposite directions with respect to the true value of parameter ω_f may be a step towards rationalizing empirical conflicts.[14]

[14] The finding that biases are in opposite directions seems to be specific to the present context, where the expectation term in the equation of interest relies on the lead of the LHS variable. In the alternative case where the expectation term is the lead of another variable, such a result is less likely to hold. This issue is left for further research.

3.2 Mis-specification of type II: Omitted dynamics

3.2.1 Analytical results

We now consider mis-specification stemming from omitted dynamics. To illustrate this case, the true DGP is assumed to include two lags of the dependent variable:

$$\begin{aligned} Y_t &= \omega_f E_t Y_{t+1} + \omega_b^1 Y_{t-1} + \left(1 - \omega_f - \omega_b^1\right) Y_{t-2} + \beta Z_t + \varepsilon_t \\ Z_t &= \rho Z_{t-1} + u_t, \end{aligned} \tag{17}$$

where the sum of parameters pertaining to lags and lead of inflation is equal to one. The reduced form of this model is thus

$$Y_t = \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \theta Z_t + \tilde{\varepsilon}_t, \tag{18}$$

where parameters φ_1 and φ_2 are related to the roots of the characteristic polynomial given by $\left(1 - \omega_f L^{-1} - \omega_b^1 L - \left(1 - \omega_f - \omega_b^1\right) L^2\right)$. The roots are given by $\varphi_1 = (1 - \omega_f)/\omega_f$, $\varphi_2 = (1 - \omega_f - \omega_b^1)/\omega_f$, and $\varphi_3 = 1$. In addition, we have $\theta = \beta / \left(\omega_f (1 - \rho)\right)$, and $\tilde{\varepsilon}_t = \varepsilon_t / \omega_f$ with $\tilde{\sigma}_\varepsilon^2 = \sigma_\varepsilon^2 / \omega_f^2$. Stationarity conditions of an AR(2) process such as in equation (18) (without forcing variable) are known to be $1 - \varphi_1 - \varphi_2 > 0$, $1 + \varphi_1 - \varphi_2 > 0$, and $\varphi_2 > -1$ (see e.g. Hamilton, 1994, p. 17–18). These conditions are equivalent to the following conditions for the structural parameters $\left(2 - 3\omega_f - \omega_b^1\right) < 0$ and $-\omega_f < \omega_b^1 < 1$ in addition to $|\rho| < 1$.

Figure 1 displays, in the plane $\{\omega_f, \omega_b^1\}$, the shaded area where the reduced-form equation (18) is stationary. In addition, to be consistent with the theoretical derivation of the hybrid model, we maintain the assumption that $\omega_f \leq 1$ (or, equivalently, $\varphi_1 \geq 0$). This yields the triangular area ABC. The segment BC corresponds to the case of non-stationarity with $\varphi_1 + \varphi_2 = 1$. Along this segment, ω_b^2 decreases from 1 to $-1/3$. The segment AB is associated with $\omega_f = 1$. This segment allows degrees of persistence $\varphi_1 + \varphi_2$ ranging from -1 (when $\omega_b^1 = 1$ and $\omega_b^2 = -1$) to 1 (when $\omega_b^1 = -1$ and $\omega_b^2 = 1$). Notice that choosing $\omega_b^1 = 1$ does not imply any restriction upon persistence of Y_t, since ω_f may itself vary. This translates to the segment CA being consistent with $\varphi_1 + \varphi_2$ ranging from 1 to -1 (ω_b^2 ranges from $-1/3$ to -1). Finally, the assumption that ω_f, ω_b^1, and ω_b^2 are all in $[0, 1]$ would imply to restrict to the area DEF defined by $\left(\omega_f, \omega_b^1\right) = \{(1, 0), (2/3, 0), (1/2, 1/2)\}$. This area is probably the most interesting one for economic interpretation.

The econometrician is assumed to erroneously select a single-lag specification, so that the estimated (mis-specified) model is a one-lag hybrid model:

$$Y_t = \alpha_f E_t Y_{t+1} + (1 - \alpha_f) Y_{t-1} + b Z_t + v_t, \tag{19}$$

along with the dynamics of the forcing variable (3). There is no mis-specification in the limiting case where $\varphi_2 = 0$, i.e. $\omega_f + \omega_b^1 = 1$.

GMM as well as ML estimators can be built quite similarly to the case with measurement error. We consider two instrument sets for GMM estimators. Estimator GMM1 is based on the instrument set $\{Y_{t-1}, Z_t\}$, while estimator GMM2 is based on the instrument set $\{Y_{t-1}, Y_{t-2}, Z_t\}$, which includes the omitted variable Y_{t-2}.

GMM1. Estimator GMM1 relies on the following moment conditions:

$$E\left[Y_{t-1}\cdot\left(Y_t-\alpha_f Y_{t+1}-(1-\alpha_f)\,Y_{t-1}-bZ_t\right)\right] = 0 \quad (20)$$

$$E\left[Z_t\cdot\left(Y_t-\alpha_f Y_{t+1}-(1-\alpha_f)\,Y_{t-1}-bZ_t\right)\right] = 0. \quad (21)$$

As in the measurement-error case, since the model is just identified, the Plims of the estimators of α_f and b are obtained by solving the two moment conditions directly. Note that the instruments are not valid in this context, since they are correlated with the error term.

GMM2. Estimator GMM2 includes $\{Y_{t-1}, Y_{t-2}, Z_t\}$ leading to an over-identified parameter set, with the property that the omitted variable appears in the instrument set. Note that GMM2 mimics actual practice of GMM of introducing several lags of the relevant variables in the instrument set. As in the case of measurement error, this estimator is built as a two-step estimator. First, Y_{t+1} is regressed on the instrument set to build the expectation of Y_{t+1} conditional on the information set, yielding $\widehat{Y}_{t+1}=(\varphi_1^2+\varphi_2)\,Y_{t-1}+\varphi_1\varphi_2 Y_{t-2}+\theta\,(\varphi_1+\rho)\,Z_t$. Then, the following equation is estimated by OLS:

$$Y_t - Y_{t-1} = \alpha_f\left(\widehat{Y}_{t+1}-Y_{t-1}\right)+bZ_t+v_t'.$$

Plims of estimators of α_f and b are thus obtained by solving the two following moment conditions:

$$E\left[\left(\widehat{Y}_{t+1}-Y_{t-1}\right)\cdot\left(Y_t-\alpha_f\widehat{Y}_{t+1}-(1-\alpha_f)\,Y_{t-1}-bZ_t\right)\right] = 0 \quad (22)$$

$$E\left[Z_t\cdot\left(Y_t-\alpha_f\widehat{Y}_{t+1}-(1-\alpha_f)\,Y_{t-1}-bZ_t\right)\right] = 0. \quad (23)$$

ML. The ML estimator is obtained by estimating the reduced form of the postulated model (19) together with equation (3), that is:

$$Y_t = \varphi Y_{t-1}+\mu Z_t+\tilde{v}_t$$

where $\varphi = (1-\alpha_f)/\alpha_f$, $\mu = b/\left(\alpha_f(1-\psi)\right)$, and $\tilde{v}_t = v_t/\alpha_f$. Parameters φ and μ are estimated by OLS, so that their Plims are given by

$$\varphi_{ML} = \frac{E\left(Z_t^2\right)E\left(Y_tY_{t-1}\right)-E\left(Y_tZ_t\right)E\left(Z_tY_{t-1}\right)}{E\left(Z_t^2\right)E\left(Y_t^2\right)-E\left(Z_tY_{t-1}\right)^2}$$

$$\mu_{ML} = \frac{E\left(Y_t^2\right)E\left(Y_tZ_t\right)-E\left(Z_tY_{t-1}\right)E\left(Y_tY_{t-1}\right)}{E\left(Z_t^2\right)E\left(Y_t^2\right)-E\left(Z_tY_{t-1}\right)^2}.$$

Then, the Plims of the ML estimators of α_f and b are given by the conditions $\alpha_f = 1/(1+\varphi)$ and $b=\mu\alpha_f(1-\rho)$.

The Plims of each estimator are summarized in Proposition 2.

Proposition 2 (Plim of estimators in case of omitted dynamics) *Let us assume that the DGP is given by equations (3) and (17). Assume that the econometrician estimates the model omitting the second lag of Y_t, corresponding to the model given by equations (3) and (19). Then, the three estimators have the following Plims:*

- *GMM estimator with instrument set* $\{Y_{t-1}, Z_t\}$ *(GMM1):*

$$
\begin{aligned}
\alpha_{GMM1} &= \left(\frac{1}{1+\varphi_1-\varphi_2}\right)\left(\frac{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\left(-1+\varphi_1-\varphi_2+\varphi_2\rho\right)\right]}{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\left(\varphi_1+\varphi_2\rho\right)\right]}\right) \\
b_{GMM1} &= \beta\left(\frac{\left(1+\varphi_1\right)\left(1-\varphi_2-\rho\left(\varphi_1+\varphi_2+\varphi_2\rho\right)\right)}{\left(1-\varphi_1\rho-\varphi_2\rho^2\right)\left(1+\varphi_1-\varphi_2\right)}\right) \\
&\quad\times\left(\frac{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\left(\varphi_1+\varphi_2\rho\right)\frac{2-\varphi_1\rho-\varphi_2\rho^2+\rho}{1-\varphi_2-\rho(\varphi_1+\varphi_2+\varphi_2\rho)}\right]}{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\left(\varphi_1+\varphi_2\rho\right)\right]}\right)
\end{aligned}
$$

- *GMM estimator with the instrument set* $\{Y_{t-1}, Y_{t-2}, Z_t\}$ *(GMM2):*

$$
\begin{aligned}
\alpha_{GMM2} &= \left(\frac{1-\varphi_1-\varphi_1\varphi_2}{1-\varphi_1^2-\varphi_1^2\varphi_2-\varphi_2}\right)\left(\frac{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\frac{-1+\varphi_1-\varphi_2+\varphi_2\rho}{1-\varphi_1-\varphi_1\varphi_2}\right]}{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\frac{-2\varphi_1\varphi_2+\varphi_2\rho-\varphi_2^2\rho}{1-\varphi_1^2-\varphi_1^2\varphi_2-\varphi_2}\right]}\right) \\
b_{GMM2} &= \beta\left(\frac{\left(1+\varphi_1\right)\left(1-\varphi_1-\varphi_2-\rho\left(1-\varphi_1-\varphi_1\varphi_2\right)\left(\varphi_1+\varphi_2+\varphi_2\rho\right)\right)}{\left(1-\varphi_1\rho-\varphi_2\rho^2\right)\left(1-\varphi_1^2-\varphi_1^2\varphi_2-\varphi_2\right)}\right) \\
&\quad\times\left(\frac{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\frac{\varphi_1(1+\rho)(1-\varphi_1-\varphi_2)+\varphi_2\rho(2+\rho-2\varphi_1\rho-\varphi_2\rho)}{1-\varphi_1-\varphi_2-\rho(1-\varphi_1-\varphi_1\varphi_2)(\varphi_1+\varphi_2+\varphi_2\rho)}\right]}{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\frac{-2\varphi_1\varphi_2+\varphi_2\rho-\varphi_2^2\rho}{1-\varphi_1^2-\varphi_1^2\varphi_2-\varphi_2}\right]}\right)
\end{aligned}
$$

- *ML estimator:*

$$
\begin{aligned}
\alpha_{ML} &= \left(\frac{1-\varphi_2}{1+\varphi_1-\varphi_2}\right)\left(\frac{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\frac{2\varphi_1+\varphi_2\rho(1-\varphi_2)}{1-\varphi_2}\right]}{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\left(1+\varphi_1+\left(1+\rho\right)\varphi_2\right)\right]}\right) \\
b_{ML} &= \beta\left(\frac{\left(1+\varphi_1\right)\left(1-\varphi_2-\rho\varphi_1\right)}{\left(1-\varphi_1\rho-\varphi_2\rho^2\right)\left(1+\varphi_1-\varphi_2\right)}\right) \\
&\quad\times\left(\frac{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\frac{2\varphi_1-\varphi_1^2\rho+\varphi_2\rho-\varphi_1\varphi_2\rho^2-\rho}{1-\varphi_2-\rho\varphi_1}\right]}{\tilde{\sigma}_\varepsilon^2+\tilde{\Lambda}^2\left[1+\varphi_2\rho\left(1+\varphi_1+\left(1+\rho\right)\varphi_2\right)\right]}\right)
\end{aligned}
$$

where $\tilde{\Lambda} = \theta\sigma_u / \left(1-\varphi_1\rho-\varphi_2\rho^2\right)$.

Proof: See the discussion above. The computation of the Plims is based on the moments and cross-moments reported in Appendix 2.

We verify that, in the case $\varphi_2 = 0$, all estimators are asymptotically unbiased, since this case imply $\alpha_{GMM1} = \alpha_{GMM2} = \alpha_{ML} = 1/\left(1+\varphi_1\right) = \omega_f$. Since we are primarily interested in the effect of omitting a lag, we consider more specifically the case $\rho = 0$, which corresponds to the forcing variable being a pure white noise. We then obtain the following corollary.

Corollary 2 *In the case* $\rho = 0$*, Plims of estimators of* α_f *and* b *are respectively given by*

$$
\begin{aligned}
\alpha_{GMM1} &= \frac{1}{1+\varphi_1-\varphi_2} \\
\alpha_{GMM2} &= \frac{1-\varphi_1-\varphi_1\varphi_2}{1-\varphi_1^2-\varphi_1^2\varphi_2-\varphi_2} \\
\alpha_{ML} &= \frac{1-\varphi_2}{1+\varphi_1-\varphi_2}
\end{aligned}
$$

and

$$\begin{aligned} b_{GMM1} &= \beta\left(\frac{(1+\varphi_1)(1-\varphi_2)}{1+\varphi_1-\varphi_2}\right) \\ b_{GMM2} &= \beta\left(\frac{(1+\varphi_1)(1-\varphi_1-\varphi_2)}{1-\varphi_1^2-\varphi_1^2\varphi_2-\varphi_2}\right) \\ b_{ML} &= \beta\left(\frac{(1+\varphi_1)(1-\varphi_2)}{1+\varphi_1-\varphi_2}\right) = b_{GMM1}. \end{aligned}$$

The following inequalities hold (provided $0 \leq \varphi_1 + \varphi_2 \leq 1$*):*

$$\begin{array}{ll} 1/2 \leq \alpha_{ML} \leq \omega_f \leq \alpha_{GMM1} \leq \alpha_{GMM2} \leq +\infty & \text{if } \varphi_2 \geq 0 \text{ (area A)} \\ 1/3 \leq \alpha_{GMM1} \leq \alpha_{GMM2} \leq \omega_f \leq \alpha_{ML} \leq 1 & \text{if } \varphi_2 \leq 0 \text{and } \varphi_1 \leq 1 \text{ (area B)} \\ 1/4 \leq \alpha_{GMM1} \leq \omega_f \leq \alpha_{GMM2} \leq \alpha_{ML} \leq 2/3 & \text{if } \varphi_2 \in [-1;\bar{\varphi}] \text{ and } \varphi_1 \geq 1 \text{ (area C)} \\ 1/4 \leq \alpha_{GMM1} \leq \omega_f \leq \alpha_{ML} \leq \alpha_{GMM2} \leq 1/2 & \text{if } \varphi_2 \in [\bar{\varphi};0] \text{ and } \varphi_1 \geq 1 \text{ (area D)} \end{array}$$

and

$$\begin{array}{ll} 0 \leq b_{GMM2} \leq b_{GMM1} = b_{ML} \leq \beta & \text{if } \varphi_2 \geq 0 \\ b_{GMM2} \leq \beta \leq b_{GMM1} = b_{ML} & \text{if } \varphi_2 \leq 0 \end{array}$$

with $\bar{\varphi} = (1-\varphi_1^2)/(1+\varphi_1^2-\varphi_1)$*, which is negative when* $\varphi_1 \geq 1$*.*

Areas A to D mentioned in Corollary 2 are depicted in **Figure 2**. In many applications, it is expected that $\varphi_2 > 0$ (i.e., $\omega_b^2 > 0$). This is the case in particular for the new Phillips curve, where additional lags are likely to have a positive cumulative effect. In this context, GMM and ML estimates are thus biased in opposite directions with respect to the true parameter ω_f. For many other cases (for instance, in the modelling of output gap), one would obtain $\varphi_2 < 0$ with $\varphi_1 > 1$. In such a context, both GMM2 and ML estimators are biased upwards with respect to the true parameter.

Result 3: *In case of omitted dynamics of the type described by equations* (3), (17), *and* (19)*, when* $\rho = 0$ *and* $\omega_f \geq 0.5$ *(corresponding to areas A and B in Figure 2), ML and GMM2 estimators of* α_f *are asymptotically biased in opposite directions with respect to the true value of parameter* ω_f*. When* $\omega_f \leq 0.5$ *(corresponding to areas C and D), ML and GMM2 estimators of* α_f *are asymptotically biased upwards with respect to the true value of parameter* ω_f*.*

Setting $\rho = 0$ may appear as an overly strong assumption, as compared with the serial correlation typically obtained for the forcing variable of some hybrid models. It is noteworthy, however, that the ranking of the Plims of ML and GMM estimators is unaltered for most parameter sets, as is confirmed by the following Result 4.

Result 4: *Whatever* $\rho \geq 0$*, for all parameter sets considered, the ranking of GMM and ML estimators is* $\alpha_{ML} \leq \alpha_{GMM1} \leq \alpha_{GMM2}$ *when* $\varphi_2 \geq 0$*, and* $\alpha_{GMM1} \leq \alpha_{GMM2} \leq \alpha_{ML}$ *when* $\varphi_2 \leq 0$ *(except for the very thin area D). Moreover the following additional result is analytically shown to hold:* $\alpha_{ML} \leq \omega_f$ *when* $\varphi_2 \geq 0$*, and* $\omega_f \leq \alpha_{ML}$ *when* $\varphi_2 \leq 0$*.*

3.2.2 Evidence and discussion

The interpretation of the ranking reported in Corollary 2 and Result 4 is analogous to the measurement-error case. Indeed, the omission of variable Y_{t-2} creates a downward bias in the GMM1 and ML estimators of the forward-looking parameter, through the correlation of Y_{t-2} with Y_{t-1}. In contrast, the GMM2 estimator suffers from an upward bias because $\widehat{Y}_{t+1}$ partly captures the effect of the omitted variable.

However, the quantitative importance of the biases is somewhat different from the case with measurement error. **Table 3** presents the Plims of GMM and ML estimators using formulae reported in Proposition 2. We select several pairs for $\{\omega_f, \omega_b^1\}$, corresponding to interesting areas in Figure 1 (the selected points are displayed in the figure). The chosen values correspond to a wide range of persistence ($\varphi_1 + \varphi_2$) of the dependent variable. We also consider $\rho = \{0; 0.5; 0.9\}$, and $\beta = \{0.1; 1\}$. We only report results when $\sigma_u/\sigma_\varepsilon = 1$, because it has been found to have only marginal effects on estimators.

The results reported in Table 3 contrast rather sharply with those reported in the case of measurement error. An important difference is that very large biases are likely to occur whatever the true value of ω_f. When the forward-looking component is very low ($\omega_f = 0.4$), with a large persistence, which mimics the results of Fuhrer (1997) for the new Phillips curve, the estimator of α_f is biased towards 0 for GMM1, but towards 1 for GMM2 and ML. Biases are rather moderate, however, since the Plim of the estimator of α_f ranges between 0.3 and 0.6, whatever the estimation procedure. On the other side, when $\omega_f = 0.8$, which is close to the results of Galí and Gertler (1999), we obtain that GMM estimators are severely biased when the missing lag has a large positive parameter. In those cases, the Plim of the estimator of α_f is as high as 1.6 for GMM1 and 2.2 for GMM2 when $\omega_b^2 = 0.5$. Contrasting with these extreme outcomes, the ML estimator displays only moderate biases.

We also notice that increasing the first-order correlation of Z_t, (ρ), induces an increase in the ML bias, but a decrease in the GMM bias. Yet, in most cases, estimators remain asymptotically biased in the opposite directions with respect to ω_f as in the case $\rho = 0$. In addition, the way an increase in ρ affects the gap between GMM and ML estimators depends on the sign of the omitted-variable parameter ω_b^2: When $\omega_b^2 < 0$, an increase in ρ results in an increase in the gap between estimators, while, conversely, when $\omega_b^2 > 0$, the gap is reduced by an increase in ρ.

In sum, the ML estimators of α_f and b are rather moderately biased even for extreme cases. This result contrasts with those obtained for the GMM estimators which are likely to display sizeable biases in cases of a large forward-looking component, when a large positive weight on the second lag is present.

4 A model with feedback

This section addresses the case when some feedback from the dependent variable towards the forcing variable is allowed. This issue is of particular interest for several reasons. First, it is likely that the dynamics of the forcing variable is related to the dependent variable. In the case of the hybrid Phillips curve, the real marginal cost depends on inflation quite naturally (see Sbordone, 2002). Also in an inflation/output-gap model, some feedback from inflation towards the output gap should be expected through the effect of the real interest rate in the I-S curve. Second, a feedback effect allows to consider a wider interval for the weight of the forward-looking component. Indeed, when Z_t is strictly exogenous, only values of ω_f larger than 0.5 are consistent with stationarity of the DGP with a single lag. When there exists a feedback from Y_{t-1} to Z_t, all values of ω_f in its authorized domain $[0;1]$ are consistent with stationarity, provided the feedback effect has a sign opposite to that of β.

4.1 The DGP

The DGP we consider is

$$
\begin{aligned}
Y_t &= \omega_f E_t Y_{t+1} + (1-\omega_f) Y_{t-1} + \beta Z_t + \varepsilon_t && (24)\\
Z_t &= \rho Z_{t-1} + \gamma Y_{t-1} + u_t, && (25)
\end{aligned}
$$

where error terms have the same properties has in model (2) and (3). In this set-up, Z_t remains predetermined in the equation for Y_t and is weakly (rather than strongly) exogenous for parameters ω_f and β. Such a feedback model cannot be solved analytically. The (autoregressive) reduced-form solution can be obtained using a numerical procedure, such as those proposed by Anderson and Moore (1985), Söderlind (1999), or Klein (2000).[15] The ML approach therefore consists, at each step of the optimization procedure, in computing the reduced form using one such procedure, and then in maximizing the log-likelihood of the reduced form. The standard GMM estimation approach still applies as such.

The aim of this section is twofold. First, we study the finite-sample biases in a correctly-specified model with feedback. As claimed above, when the forcing variable is strongly exogenous, stationarity of the DGP requires $\omega_f > 0.5$. In such a set-up, we obtained in Section 2.3 that the GMM estimator of ω_f has a negative bias in case of weak instrument relevance or instrument redundancy, while the ML estimator does not display any noticeable bias. Allowing the true parameter ω_f to be lower than 0.5 gives the opportunity to extend these previous results. Second, we aim at measuring the asymptotic bias in a mis-specified model when ω_f is smaller than the bounds above (1/2 with a single lag and 1/3 with two lags). This is an important issue, since several authors have advocated that the forward-looking component may be in fact very small (see Fuhrer, 1997, or Fuhrer and Rudebusch, 2002).

4.2 Finite-sample bias in the correctly-specified model

As in the case without feedback, we begin with a brief investigation of the finite-sample properties of GMM and ML estimators in a correctly-specified model. We select the following parameter values for our baseline case: $\omega_f = 0.25$, $\rho = 0.75$, and $\sigma_\varepsilon^2 = \sigma_u^2 = 1$. We choose $\omega_f = 0.25$ because the main interest for allowing feedback is to consider a low value of ω_f. Such a value has been found to be plausible in Fuhrer (1997) in the case of the Phillips curve. We select values of β in $\{0.1; 1\}$ and γ in $\{-0.5; -0.1\}$. The value of $\gamma = -0.5$ is taken from Fuhrer and Rudebusch (2002). It is arguably large, but our purpose is to illustrate the effect

[15] An alternative estimation procedure has been recently put forward by Kurmann (2002) in a closely related context, following the approach originally developed by Sargent (1979). This approach relies on estimating a VAR model for the two variables including the variable of interest Y_t. Yet, the VAR parameters are estimated under the constraints imposed by the rational-expectation equation (here, the hybrid equation).

of feedback towards the forcing variable. Other values considered are $\omega_f = \{0.5; 0.75\}$ and $\rho = \{0.5; 0.75; 0.9\}$.

Table 4 reports results for GMM (with 0 and 7 lags in the instrument set) and for ML, with sample size $T = 100$. We also report the Plim of OLS estimators (computed from a simulation with $T = 100,000$). For $\omega_f = 0.5$ and 0.75, we find similar results to the case without feedback, so we focus on the case $\omega_f = 0.25$ only. Our main findings are as follows.

First, the GMM bias is negligible for $\beta = 1$ and moderate for $\beta = 0.1$. This is because the GMM estimator is biased in direction of the OLS estimator. When $\beta = 0.1$, the Plim of the OLS estimator is very close to 0.5, so that the median of the GMM estimator ranges between 0.3 and 0.5. Yet, when $\beta = 1$, the Plim of OLS is much closer to 0.25, so that even OLS is nearly unbiased.

Second, in some instances, ML appears to suffer from a noticeable finite-sample bias. This bias is related to the finite-sample bias in the OLS estimator of the autoregressive parameter in an AR(1) model (see, e.g., Sawa, 1978). The bias is substantial only when the autoregressive root of the univariate process for Y_t is large, i.e. when ω_f is small. However, given the form of the nonlinear function between the AR parameter and the estimated structural parameter, a small bias in the AR root translates into a noticeable bias in α_f. ML also experiences low performance, when ρ is low, because of weak empirical identification. Indeed in this case, Z_t can hardly be distinguished from Y_{t-1}.

Last, the estimator of b is generally biased towards 0 when $\beta = 0.1$, while it is biased upwards when $\beta = 1$. These biases are moderate, however.

To sum up, the main conclusions found previously extend to this more plausible case. With GMM, the finite-sample bias in the estimator of α_f is generally positive. It is worth emphasizing, however, that the parameter is biased towards 0.5 and not towards 1. Therefore, it cannot account for the contrast between GMM and ML estimates reported in the literature.

4.3 Asymptotic bias in the case of mis-specification

We now concentrate on the size of the bias in GMM and ML estimators in case of mis-specification. We consider measurement error and omitted dynamics in turn.

4.3.1 Measurement error

In case of measurement error, the assumed DGP is given by equations (24), (25), and (8), which we repeat for reader's convenience:

$$\begin{aligned} Y_t &= \omega_f E_t Y_{t+1} + (1 - \omega_f) Y_{t-1} + \beta Z_t + \varepsilon_t \\ Z_t &= \rho Z_{t-1} + \gamma Y_{t-1} + u_t \\ X_t &= a Z_t + e_t, \end{aligned}$$

while the econometrician erroneously estimates

$$\begin{aligned} Y_t &= \alpha_f E_t Y_{t+1} + (1 - \alpha_f) Y_{t-1} + b X_t + v_t \\ X_t &= \psi X_{t-1} + \kappa Y_{t-1} + w_t. \end{aligned}$$

The baseline parameters are, as previously, $\omega_f = 0.25$, $\rho = 0.75$, and $\sigma_u/\sigma_\varepsilon = 1$. We also select values of γ in $\{-0.5; -0.1\}$, β in $\{0.1; 1\}$, and τ in $\{0.1; 0.5; 0.9\}$. Since asymptotic biases cannot be computed analytically in this set-up, they are computed using a large Monte-Carlo simulation ($T = 25,000$) for each experiment.

Table 5 reports the results of these simulations. As a preliminary comment, it is worth emphasizing that, unlike the model without feedback of Section 3, both GMM and ML estimators are now biased *upwards* when the forward-looking component is chosen to be very low ($\omega_f = 0.25$). The bias in α_f is moderate when the quality of the proxy, τ, is large, with a bias smaller than 0.1. In contrast, when τ is chosen to be small ($\tau = 0.1$), we obtain sizeable biases, as high as 0.3 for the GMM as well as the ML estimators. By and large, biases are found to be located in a similar range for the three estimators. As expected, biases are much smaller when the forward-looking parameter ω_f is equal to 0.5 or 0.75.

In all cases, the estimator of the parameter of the forcing variable (b) is found to be biased towards 0. The bias decreases with the quality τ of the proxy.

4.3.2 Omitted dynamics

In case of omitted dynamics, the DGP is given by:

$$\begin{aligned} Y_t &= \omega_f E_t Y_{t+1} + \omega_b^1 Y_{t-1} + \left(1 - \omega_f - \omega_b^1\right) Y_{t-2} + \beta Z_t + \varepsilon_t \\ Z_t &= \rho Z_{t-1} + \gamma Y_{t-1} + u_t, \end{aligned}$$

while the econometrician estimates the (mis-specified) model given by equations (24) and (25).

The baseline parameters are unchanged, except for the weight of the lag and lead components $(\omega_f, \omega_b^1, \omega_b^2)$, for which we consider several different sets, with $(0.65, 0.35, 0)$ corresponding to the case with no mis-specification.

Table 6 reports the results of these simulations. Interestingly, we notice that biases are very substantial for GMM estimators, even when the weight of the omitted lag is low. (An exception being the case with no mis-specification). For instance, when $\omega_f = 0.5$, $\omega_b^1 = 0.25$, and $\omega_b^2 = 0.25$, we obtain a Plim of the estimator of α_f equal to 0.97 for GMM2, and 0.58 for ML. The Plim of GMM estimator is often higher than 1, even when $\omega_f \leq 0.5$, so that the econometrician would reject the hybrid model in favor of a purely forward-looking model. In addition, biases are particularly large for the GMM2 estimator. The Plim of the estimator α_f is found to be larger than 1 in all instances but two cases, corresponding to the lowest weights on the second lag. These results also provide an extension to Corollary 2: The ML bias is in most cases much smaller than the GMM bias. The ranking of Plims of estimators with respect to the true value of parameter ω_f does not hold anymore, however.

When the absolute value of the feedback parameter γ decreases, the GMM bias increases systematically. The case $\beta = 1$ helps to reduce persistence in the system, so that biases are in general much lower than for $\beta = 0.1$.

A further interesting result is that the Plim of the GMM estimator of b is found to be negative in many instances. This is the case in particular when the weight ω_b^2 is positive and large and when the parameter of the forcing variable is small ($\beta = 0.1$). Also the bias in b is much larger for GMM2 than for GMM1.

Result 5: *In a model with feedback, measurement error is very unlikely to fill the gap between parameter estimates obtained by GMM and ML. In contrast, a limited amount of mis-specification in the form of an omitted dynamics may produce discrepancies between GMM and ML estimators of the degree of forward-lookingness that are in excess of* 0.5.

5 Rationalizing evidence on the hybrid Phillips curve

This section illustrates results obtained in the previous sections, using them to reexamine estimates of the New Keynesian Phillips curve.

5.1 Estimating the hybrid Phillips curve

The baseline hybrid inflation model is the following:

$$\pi_t = \omega_f E_t \pi_{t+1} + \omega_b \pi_{t-1} + \beta Z_t + \varepsilon_t \tag{26}$$

where π_t is the inflation rate and Z_t is the real marginal cost. Parameters ω_f and ω_b are positive with $\omega_f + \omega_b \leq 1$. This model has been proposed originally by Chadha, Masson, and Meredith (1992). Two proxies for real marginal cost have been considered in the literature: the output gap and the real ULC. With output gap as a forcing variable, this model nests as special cases the traditional Phillips curve ($\omega_f = 0$), the Taylor (1980) forward-looking Phillips curve ($\omega_f = 1$), and the Fuhrer and Moore (1995) model with two-period contracts ($\omega_f = 1/2$). With real ULC as a forcing variable, this model is very close to the hybrid Phillips curve put forward by Galí and Gertler (1999) or Christiano, Eichenbaum, and Evans (2001).

Although several recent work has focused on the real ULC instead of output gap as a proxy for the real marginal cost, we resort to the traditional forcing variable, because it provides a more interesting illustration of our theoretical results. In particular, the real ULC was not found to be caused by inflation (cf. also Kurmann, 2002), while output gap has long been found to be related to inflation (see Rudebusch, 2002). The dynamics of the output gap is adequately modelled with three own lags and three lags of inflation.[16] We thus consider equation (26) together with:

$$Z_t = \sum_{i=1}^{3} \rho_i Z_{t-i} + \sum_{i=1}^{3} \gamma_i \pi_{t-i} + u_t. \tag{27}$$

As indicated previously, Galí and Gertler (1999) have derived the hybrid Phillips curve assuming that some firms set their price optimally in a sticky-price framework. Following Calvo (1983), only a fraction of firms is allowed to reset their price, at each date. Some of these firms use a backward-looking rule of thumb, while the others are forward-looking. In such a context, the sum of the backward and forward-looking terms lies theoretically between δ and 1, where δ is the discount factor. Therefore, $\omega_f + \omega_b$ should be very close to 1 for any

[16] A standard modelling of the output-gap dynamics typically incorporates the real interest rate instead of inflation. Introducing an additional equation corresponding to the monethary authorities' reaction function would, however, veil the interpretation of our empirical evidence in the light of our theoretical results. Our specification (27) may therefore be interpreted as a reduced-form equation.

plausible value of δ. In the application below, we impose $\omega_f + \omega_b = 0.99$ in equation (26), a restriction which is very close to the one adopted by Galí and Gertler ($\delta = 0.99$).

We define $\tilde{W}_t = (\pi_t, Z_t)'$ and $\eta_t = (\varepsilon_t, u_t)'$. As previously, innovations η_t are assumed to be serially uncorrelated, with $E(\eta_t \eta_{t'}') = \Sigma$ if $t' = t$ and 0 otherwise. Yet, the contemporaneous covariance matrix Σ is allowed to have a non-zero covariance between ε_t and u_t, denoted $\sigma_{\varepsilon u}$. Unknown parameters are gathered together in $\xi = \{\omega_f, \beta, \mu_z, \rho_1, \rho_2, \rho_3, \gamma_1, \gamma_2, \gamma_3, \sigma_\varepsilon^2, \sigma_u^2, \sigma_{\varepsilon u}\}$.

As regards GMM and ML estimation procedures, we consider slight departures from the framework described in previous sections, in order to follow the standards of the empirical literature. Consistently with the non-zero covariance between ε_t and u_t, valid instruments are dated $t-1$ or earlier only. For estimator GMM2, the instrument set contains $\left\{\tilde{W}_{t-1}, \tilde{W}_{t-2}, \tilde{W}_{t-3}\right\}$, while GMM1 estimator is obtained when additional lags of inflation (beyond π_{t-1}) are omitted. Following the approach adopted by Galí and Gertler (1999), we compute the GMM weighting matrix using the Newey and West (1987) procedure with a bandwidth of 4 lags.

Concerning the ML approach, equations (26) and (27) are estimated simultaneously, using the procedure developed by Anderson and Moore (1985) to compute the reduced form of the model

$$\tilde{W}_t = \sum_{j=1}^{3} B_j \tilde{W}_{t-j} + B_0 \eta_t, \tag{28}$$

where B_0 is the matrix of contemporaneous coefficients. Therefore, the concentrated log-likelihood function for sample $\left\{\tilde{W}_t\right\}_{t=1}^{T}$ is defined as follows:

$$\ln L(\xi) = -T\left[1 + \ln(2\pi)\right] - \frac{T}{2}\ln\left|\hat{\Sigma}\right| + \frac{T}{2}\ln\left|\hat{B}_0^{-1}\right|^2$$

where $\hat{\Sigma} = \frac{1}{T}\sum_{t=1}^{T}\hat{\eta}_t(\xi)\,\hat{\eta}_t(\xi)'$ is the estimated covariance matrix of residuals. The log-likelihood function is maximized using the BFGS algorithm of the GAUSS Constrained Maximum Likelihood package procedure.

Estimation is performed on US data, over the sample period 1960:I-2000:IV. Inflation is defined as the annualized quarterly percent change in the implicit GDP deflator. The output-gap measure computed as the deviation of GDP from a trend with a break in slope in 1973:IV. Empirical estimates of the baseline hybrid model are reported in **Table 7**.

In spite of some differences in sample period and instrument set, results in the table are broadly in accordance with previous estimates. The ML estimation provides a significant impact of output gap (with the expected positive sign) and the backward-looking component is dominant. The estimate of the forward-looking parameter ($\omega_f = 0.45$) is larger than the parameter obtained by Fuhrer, since his estimates of ω_f range from 0.02 to 0.20. This evidence contrasts quite sharply with estimates performed with GMM which both point to a dominant forward-looking component ($\omega_f = 0.66$ for GMM1 and 0.70 for GMM2). In addition, GMM estimates suggest a negative, yet non-significant, parameter of output gap.

Overall, our estimations reveal that, with our data and our mere framework, the gap between GMM2 and ML estimates of ω_f is as high as 0.25. Such a contrast across estimation methods has already been found by Jondeau and Le Bihan (2001) and Lindé (2001).

Finally, residual check does not point to a clear-cut mis-specification. On one hand, the Hansen J-statistic for over-identifying restrictions does not reject the null hypothesis for the

GMM estimations, while, on the other hand, statistics for ML residuals reveal a slight serial correlation of the Phillips-curve residuals. In addition, the positive gap between GMM2 and ML estimates of ω_f is very suggestive of a missing lag, as indicated in previous sections.[17]

5.2 Towards filling the gap

In order to investigate the consequences of omitted dynamics, we consider now the following DGP in which inflation depends on several own lags:

$$\pi_t = \omega_f E_t \pi_{t+1} + \sum_{i=1}^{3} \omega_b^i \pi_{t-i} + \beta Z_t + \varepsilon_t \quad (29)$$

$$Z_t = \sum_{i=1}^{3} \rho_i Z_{t-i} + \sum_{i=1}^{3} \gamma_i \pi_{t-i} + u_t, \quad (30)$$

where the restricted model (with omitted dynamics) is obtained by setting $\omega_b^2 = \omega_b^3 = 0$.[18] As indicated in Section 3, such a model has been considered in several studies. For instance, Rudebusch (2002), Mavroeidis (2001), and Galí, Gertler, and López-Salido (2001) estimate such a model, while Fuhrer (1997), Coenen and Wieland (2000), Roberts (2001), or Kozicki and Tinsley (2002) introduce, in addition, leads of inflation. Kozicki and Tinsley (2002) also review several interpretations of such a specification.

Table 8 reports ML estimates of model (29) and (30). It indicates that the weight of the forward-looking component is as low as 0.4. The third lag is also found to be significant. The serial correlation of residuals found in the model with a single lag has disappeared, although the serial correlation of squared residuals remains.

We now investigate the source of the discrepancy between the two estimation methods using some Monte-Carlo simulations. Our previous experiments in Sections 3 and 4 have highlighted that only a moderate value of ω_f with an omitted dynamics can help to reconcile the Plims of estimators with empirical evidence. In this case, indeed, we would obtain a significant bias in GMM as well as ML estimators, but the GMM estimator would be more biased towards 1 than the ML estimator. Therefore, we assume that the ML estimation of the model with three lags of inflation provides a rough description of the true DGP. The DGP used in Monte-Carlo simulations is thus the model (29) and (30) with the parameter estimates reported in Table 8. We simulate 2000 samples of size $T = 160$ of this DGP. Then, we estimate the model with one single lag and lead using GMM as well as ML estimation procedures. If the true DGP is close to our simulated DGP, we expect the parameter estimates to be close to those found on US data.

Table 9 reports the median and the MAD of the finite-sample properties of the estimators. The ranking obtained for the three estimators is in accordance with the theoretical analysis as well as with the empirical results presented above. As indicated in Result 4 in the case without

[17] We performed the same exercice with real ULC in place of output gap as forcing variable. We obtained very similar results. In particular, we found a discrepancy between GMM2 and ML estimators as high as 0.17.

[18] We illustrate our theoretical results focusing on a mis-specification due to omitted dynamics instead of measurement error for two reasons. First, the gap between GMM and ML estimators of ω_f was found to be larger in the former case than in the latter case. Second, it is much more convenient to specify a DGP with additional lags: Lags are readily introduced in the assumed DGP, while a plausible measurement error would be much more difficult to design.

feedback (Section 3), we obtain the ranking $\alpha_{ML} < \alpha_{GMM1} < \alpha_{GMM2}$. The discrepancy between α_{ML} and α_{GMM2} is as high as 0.15, suggesting that the gap found on US data may well be due, to a great extent, to omitted dynamics. The fact that the three estimators are found to be larger than the true value of the parameter ($\omega_f = 0.4$) may be explained by the feedback effect of the inflation rate towards output gap. This is consistent with Monte-Carlo simulations presented in Section 4.

Finally, it should be noticed that omitted dynamics cannot explained the whole discrepancy between GMM2 and ML estimators found with historical data. This may be explained by a number of reasons. In particular, there may be, in addition, a measurement error in the forcing variable, which would exacerbate the discrepancy. Also, departure from iidness and/or normality may partly fill the unexplained gap.

6 Conclusion

This paper has analyzed the properties of GMM and ML estimators in hybrid models. Our motivation was the gap between the large degree of forward-looking behavior typically found when implementing GMM and the low degree of forward-lookingness obtained by ML. Our findings can be summarized as follows. First, finite-sample biases are not able to fill the gap between empirical estimates. The GMM bias is small unless a large number of redundant variables are used. Furthermore, the bias is towards the Plim of the OLS which is typically close to 0.5, and the estimator is biased towards a lower value than the ML estimator in finite sample.

Second, plausible mis-specifications can produce substantial differences between the two estimators. In particular, in case of measurement error, GMM can be moderately biased towards 1. Analytical results establish that, in a simple model with a strongly exogenous forcing variable, asymptotic GMM and ML biases of the degree of forward-lookingness are in opposite directions with respect to the true value of the parameter.

In case of omitted dynamic, the GMM estimator is likely to be severely biased towards very large values in case of a large forward-looking components. In many plausible cases, biases of GMM and ML point to opposite directions. While this latter property does not carry on necessarily to more elaborate models with feedback, we still find, in the case of mis-specification in models with feedback, that GMM is generally more widely biased than ML in a way that is likely to fill the gap between empirical estimates.

Results in the present paper point to one critical source of the discrepancy in estimators of a hybrid equation: That a relevant forcing variable is omitted from the estimated equation but included in the GMM instrument set. Such an instance, rather likely if a large number of instruments is used, will cause the lead of the dependent variable to capture the effect of the omitted variable, and its parameter to be over-estimated. In the type of set-up analyzed here, mis-specification of the equation of interest is typically found to be more harmful to the GMM estimator than to the ML estimator. This finding to some extent balances the well-known fact that in rational-expectation models, ML may, unlike GMM, suffer from mis-specification of the auxiliary model.

Our theoretical results are used to rationalize differences in estimates of the hybrid Phillips curve found in the literature. Small mis-specification (such as omitting one relevant lag, even with a modest parameter value, in the inflation dynamics) turns out to imply substantial over-estimation of the degree of forward-lookingness found by the GMM estimator. The ML appears to be much less severely biased in this context.

Results in this paper may be extended in several directions. First, they suggest that a test for mis-specification in the hybrid model may be based on the difference between ML and GMM2 estimates, since the two estimators are typically biased in opposite directions with respect to the true value of the forward-looking parameter. Second, future research could investigate whether the motivation for discrepancy in estimators outlined here is also relevant in other empirical applications.

Appendices

Appendix 1: Moments and cross-moments in the model with a single lag

This Appendix reports several moment and cross-moment coefficients implied by the DGP (2), (3), and (8). These results are useful to compute the moment conditions. Moments of Z_t and cross-moments with Y_t are:

$$\begin{aligned} E\left(Z_t^2\right) &= \frac{\sigma_u^2}{1-\rho^2} = \sigma_Z^2 \\ E\left(Z_t Z_{t-i}\right) &= \rho^i \sigma_Z^2 \\ E\left(Z_t Y_t\right) &= \frac{\theta\sigma_Z^2}{1-\varphi_1\rho} = \Gamma_0 \\ E\left(Z_t Y_{t-1}\right) &= \rho\Gamma_0 \\ E\left(Y_t Z_{t-1}\right) &= \varphi_1\Gamma_0 + \theta\rho\sigma_Z^2. \end{aligned}$$

Moments involving Y_t only are:

$$\begin{aligned} E\left(Y_t^2\right) &= \frac{\tilde{\sigma}_\varepsilon^2}{1-\varphi_1^2} + \frac{1+\varphi_1\rho}{1-\varphi_1^2}\theta\Gamma_0 = \Phi_0 \\ E\left(Y_t Y_{t-1}\right) &= \varphi_1\Phi_0 + \theta\rho\Gamma_0 \\ E\left(Y_t Y_{t-2}\right) &= \varphi_1^2\Phi_0 + \theta\rho\left(\varphi_1+\rho\right)\Gamma_0. \end{aligned}$$

Finally, moments of X_t and cross-moments with Y_t are:

$$\begin{aligned} E\left(X_t^2\right) &= a^2\sigma_Z^2 + \sigma_e^2 \\ E\left(X_t X_{t-1}\right) &= \rho a^2 \sigma_Z^2 \\ E\left(X_t Y_{t-i}\right) &= aE\left(Z_t Y_{t-i}\right) \qquad \forall i. \end{aligned}$$

Appendix 2: Moments and cross-moments in the model with two lags

Moments of Z_t are the same as in Appendix 1, while cross-moments with Y_t are:

$$\begin{aligned} E\left(Z_t Y_t\right) &= \frac{\theta\sigma_Z^2}{1-\varphi_1\rho-\varphi_2\rho^2} = \tilde{\Gamma}_0 \\ E\left(Z_t Y_{t-i}\right) &= \rho^i\tilde{\Gamma}_0, \text{ for all } i>0 \\ E\left(Y_t Z_{t-1}\right) &= \left(\varphi_1+\varphi_2\rho\right)\tilde{\Gamma}_0 + \theta\rho\sigma_Z^2. \end{aligned}$$

Moments involving Y_t only are:

$$\begin{aligned} E\left(Y_t^2\right) &= \frac{\left(1-\varphi_2\right)\tilde{\sigma}_\varepsilon^2 + \left[\varphi_1\rho\left(1+\varphi_2\right) + \left(1-\varphi_2\right)\left(1+\varphi_2\rho^2\right)\right]\theta\tilde{\Gamma}_0}{\left(1+\varphi_2\right)\left(1-\varphi_1-\varphi_2\right)\left(1+\varphi_1-\varphi_2\right)} = \tilde{\Phi}_0 \\ E\left(Y_t Y_{t-1}\right) &= \frac{\varphi_1}{1-\varphi_2}\tilde{\Phi}_0 + \frac{\theta\rho}{1-\varphi_2}\tilde{\Gamma}_0 \\ E\left(Y_t Y_{t-2}\right) &= \left(\varphi_2 + \frac{\varphi_1^2}{1-\varphi_2}\right)\tilde{\Phi}_0 + \left(\frac{\varphi_1}{1-\varphi_2}+\rho\right)\theta\rho\tilde{\Gamma}_0. \end{aligned}$$

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Captions

Table 1: This table reports summary statistics on the finite-sample distribution of the estimators of the model with a single lag. Parameter sets are $\omega_f = \{0.55; 0.75; 0.95\}$, $\rho = \{0.1; 0.5; 0.9\}$, $\beta = \{0.1; 1\}$, and $\sigma_\varepsilon = \sigma_u = 1$. The median and the MAD of the estimator distribution are computed over 2000 samples of size $T = 100$. Estimation methods are GMM (with $L = 0$ and 7 lags of $W_t = \{Y_{t-1}, Z_t\}$ as instrument sets) and ML. The Plim of the OLS estimator is also computed, with a sample of $100,000$ observations.

Table 2: This table reports the Plim of GMM and ML estimators in the case of measurement error. Parameter sets are $\omega_f = \{0.55; 0.75; 0.95\}$, $\rho = \{0.1; 0.5; 0.9\}$, $\tau = \{0.1; 0.5; 0.9\}$, $\beta = \{0.1; 1\}$, $a = 1$, and $\sigma_\varepsilon = \sigma_u = 1$. The Plims are computed using Proposition 1. Bounds for the Plims of estimators of α_f are obtained for $\omega_f = \{0.5; 1\}$, $\rho = \{0; 1\}$, and $\tau = \sigma_\varepsilon = 0$.

Table 3: This table reports the Plim of GMM and ML estimators in the case of omitted dynamics. Several pairs of $\{\omega_f, \omega_b^1\}$ are selected, as displayed in Figure 1. Other parameter sets are $\rho = \{0; 0.5; 0.9\}$, $\beta = \{0.1; 1\}$, $\sigma_\varepsilon = \sigma_u = 1$. The Plims are computed using Proposition 2. Bounds for the Plims of estimators of α_f are obtained for $\{\omega_f; \omega_b^1\} = \{A; B; C\}$, $\rho = \{0; 1\}$, and $\sigma_\varepsilon = 0$.

Table 4: This table reports summary statistics on the finite-sample distribution of the estimators in the feedback model with a single lag. The baseline parameter set is given by $\omega_f = 0.25$, $\rho = 0.75$, $\beta = \{0.1; 1\}$, $\gamma = \{-0.5; -0.1\}$, and $\sigma_\varepsilon = \sigma_u = 1$. Other values considered are $\omega_f = \{0.5; 0.75\}$, and $\rho = \{0.5; 0.9\}$. The median and the MAD of the parameter distribution are computed over 2000 samples of size $T = 100$. Estimation methods are GMM (with $L = 0$ and 7 lags of $W_t = \{Y_{t-1}, Z_t\}$ as instrument sets) and ML. The Plim of the OLS estimator is also computed, with a sample of $100,000$ observations.

Table 5: This table reports the Plim of GMM and ML estimators in the feedback model with measurement error. The baseline parameter set is given by: $\omega_f = 0.25$, $\rho = 0.75$, $\beta = \{0.1; 1\}$, $\gamma = \{-0.5; -0.1\}$, $\tau = \{0.1; 0.5; 0.9\}$, and $\sigma_\varepsilon = \sigma_u = 1$. Other values considered are $\omega_f = \{0.5; 0.75\}$, and $\rho = \{0.5; 0.9\}$. The Plims are computed with a sample of $25,000$ observations.

Table 6: This table reports the Plim of GMM and ML estimators in the feedback model with omitted dynamics. The baseline parameter set is given by: $\omega_f = 0.33$, $\omega_b^1 = 0.33$, $\rho = \{0.5; 0.75; 0.9\}$, $\beta = \{0.1; 1\}$, and $\gamma = \{-0.5; -0.1\}$, and $\sigma_\varepsilon = \sigma_u = 1$. Other values considered are $\omega_f = \{0.25; 0.50; 0.65; 0.75\}$. The Plims are computed with a sample of $25,000$ observations.

Table 7: This table reports GMM and ML estimates of the hybrid Phillips curve with a single lag over the period 1960:I-2000:IV. The instrument sets are $\{Y_{t-1}, Z_{t-1}, Z_{t-2}, Z_{t-3}\}$ for GMM1 and $\{Y_{t-1}, Y_{t-2}, Y_{t-3}, Z_{t-1}, Z_{t-2}, Z_{t-3}\}$ for GMM2. J-stat denotes the Hansen's statistic for the test of over-identifying restrictions, lnL is the sample log-likelihood and see is the standard error of estimates. $Q(4)$ is the Ljung-Box statistic for the test that the first 4 serial correlations of residuals are jointly zero. $R(1)$ is the Engle statistic for the test that the

first serial correlation of squared residuals is zero. J-B is the Jarque-Bera test statistic for the null of normality.

Table 8: This table reports ML estimates of the hybrid Phillips curve with three lags over the period 1960:I-2000:IV. lnL is the sample log-likelihood and see is the standard error of estimates. $Q(4)$ is the Ljung-Box statistic for the test that the first 4 serial correlations of residuals are jointly zero. $R(1)$ is the Engle statistic for the test that the first serial correlation of squared residuals is zero. J-B is the Jarque-Bera test statistic for the null of normality.

Table 9: This table reports summary statistics on the finite-sample distribution of the GMM and ML estimators of the model with a single lag when the DGP is in fact a hybrid model with three lags. The DGP corresponds to the model (29) and (30) with parameters reported in Table 8. The finite-sample distribution is obtained using Monte-Carlo simulations with 2000 samples of size $T = 160$. The distribution is summarized using the median and the MAD.

Figure 1: This figure displays, in the plane $\{\omega_f, \omega_b^1\}$, the domain of validity of the hybrid model with two lags. The shaded area corresponds to the domain of stationarity of an AR(2) process. The area ABC corresponds to the additional constraint that $\omega_f \leq 1$. The are DEF corresponds to the domain where $0 \leq \omega_f, \omega_b^1, \omega_b^2 \leq 1$. The six points in the figure are the pairs $\{\omega_f, \omega_b^1\}$ selected for Table 3. Last, we denote $\omega = \{\omega_f, \omega_b^1, \omega_b^2\}$ and $\varphi = \{\varphi_1, \varphi_2\}$.

Figure 2: This figure displays, in the plane $\{\omega_f, \omega_b^1\}$, the areas corresponding to the different rankings between Plims of GMM and ML estimators, as reported in Corollary 2.

Table 1: Finite-sample properties of estimators in the model with a single lag

Struct. parameters		Statistic	Panel A: β=0.1								Panel B: β=1							
			GMM (L=0)		GMM (L=7)		ML		Plim OLS		GMM (L=0)		GMM (L=7)		ML		Plim OLS	
ω_f	ρ		ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β
0.55	0.1	median	0.55	0.10	0.52	0.10	0.55	0.10	0.50	0.11	0.54	1.01	0.42	1.24	0.55	1.00	0.34	1.36
		MAD	*0.06*	*0.08*	*0.06*	*0.09*	*0.02*	*0.05*			*0.08*	*0.18*	*0.07*	*0.20*	*0.01*	*0.12*		
0.55	0.5	median	0.55	0.10	0.52	0.12	0.55	0.10	0.49	0.12	0.54	1.05	0.47	1.32	0.55	1.01	0.32	1.88
		MAD	*0.06*	*0.05*	*0.06*	*0.07*	*0.01*	*0.03*			*0.06*	*0.28*	*0.06*	*0.34*	*0.00*	*0.18*		
0.55	0.9	median	0.55	0.11	0.53	0.14	0.55	0.11	0.47	0.19	0.55	1.15	0.53	1.38	0.55	1.11	0.46	2.02
		MAD	*0.03*	*0.06*	*0.03*	*0.07*	*0.00*	*0.04*			*0.03*	*0.58*	*0.03*	*0.72*	*0.00*	*0.47*		
0.75	0.1	median	0.75	0.11	0.68	0.11	0.75	0.10	0.50	0.12	0.75	1.01	0.65	1.05	0.75	1.00	0.43	1.17
		MAD	*0.09*	*0.10*	*0.08*	*0.12*	*0.06*	*0.07*			*0.08*	*0.15*	*0.07*	*0.16*	*0.03*	*0.14*		
0.75	0.5	median	0.75	0.10	0.68	0.11	0.75	0.10	0.50	0.12	0.74	1.01	0.66	1.08	0.75	1.01	0.44	1.25
		MAD	*0.09*	*0.07*	*0.08*	*0.08*	*0.05*	*0.04*			*0.07*	*0.18*	*0.06*	*0.22*	*0.02*	*0.18*		
0.75	0.9	median	0.74	0.11	0.68	0.12	0.75	0.11	0.49	0.13	0.75	1.07	0.69	1.16	0.75	1.07	0.48	1.33
		MAD	*0.06*	*0.05*	*0.06*	*0.06*	*0.03*	*0.05*			*0.05*	*0.44*	*0.05*	*0.53*	*0.00*	*0.44*		
0.95	0.1	median	0.95	0.09	0.83	0.10	0.95	0.10	0.50	0.11	0.94	1.00	0.82	1.01	0.95	1.00	0.49	1.03
		MAD	*0.13*	*0.13*	*0.12*	*0.14*	*0.09*	*0.09*			*0.11*	*0.17*	*0.10*	*0.19*	*0.06*	*0.16*		
0.95	0.5	median	0.95	0.10	0.83	0.10	0.96	0.10	0.50	0.11	0.94	1.02	0.82	1.03	0.95	1.02	0.49	1.04
		MAD	*0.13*	*0.08*	*0.11*	*0.10*	*0.09*	*0.05*			*0.10*	*0.19*	*0.09*	*0.21*	*0.04*	*0.18*		
0.95	0.9	median	0.94	0.11	0.83	0.11	0.95	0.11	0.50	0.10	0.94	1.09	0.83	1.10	0.95	1.10	0.50	1.06
		MAD	*0.12*	*0.05*	*0.10*	*0.06*	*0.06*	*0.05*			*0.09*	*0.45*	*0.08*	*0.50*	*0.01*	*0.44*		

Table 2: Plim of estimators in the case of measurement error

Structural parameters			Panel A: β=0.1						Panel B: β=1					
			GMM1		GMM2		ML		GMM1		GMM2		ML	
ω_f	ρ	τ	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b
0.55	0.10	0.10	0.55	0.01	0.56	0.01	0.55	0.01	0.52	0.10	0.84	0.05	0.55	0.11
0.55	0.10	0.50	0.55	0.05	0.56	0.05	0.55	0.05	0.54	0.51	0.76	0.31	0.55	0.52
0.55	0.10	0.90	0.55	0.09	0.55	0.09	0.55	0.09	0.55	0.90	0.61	0.80	0.55	0.91
0.55	0.50	0.10	0.54	0.01	0.58	0.01	0.55	0.02	0.41	0.15	0.70	0.04	0.52	0.17
0.55	0.50	0.50	0.54	0.05	0.57	0.05	0.55	0.07	0.49	0.62	0.66	0.29	0.53	0.69
0.55	0.50	0.90	0.55	0.09	0.55	0.09	0.55	0.10	0.54	0.94	0.58	0.79	0.55	0.97
0.55	0.90	0.10	0.40	0.03	0.57	0.01	0.51	0.04	0.37	0.30	0.57	0.07	0.50	0.37
0.55	0.90	0.50	0.50	0.08	0.56	0.04	0.52	0.16	0.50	0.79	0.57	0.42	0.52	1.51
0.55	0.90	0.90	0.54	0.10	0.55	0.09	0.54	0.15	0.54	0.97	0.55	0.87	0.54	1.47
0.75	0.10	0.10	0.75	0.01	0.75	0.01	0.75	0.01	0.72	0.10	0.87	0.09	0.73	0.11
0.75	0.10	0.50	0.75	0.05	0.75	0.05	0.75	0.05	0.73	0.50	0.82	0.48	0.74	0.52
0.75	0.10	0.90	0.75	0.09	0.75	0.09	0.75	0.09	0.75	0.90	0.76	0.89	0.75	0.91
0.75	0.50	0.10	0.74	0.01	0.76	0.01	0.74	0.02	0.57	0.11	0.83	0.09	0.61	0.13
0.75	0.50	0.50	0.74	0.05	0.75	0.05	0.74	0.07	0.65	0.54	0.80	0.48	0.66	0.58
0.75	0.50	0.90	0.75	0.09	0.75	0.09	0.75	0.10	0.73	0.91	0.76	0.89	0.73	0.94
0.75	0.90	0.10	0.57	0.01	0.76	0.01	0.53	0.02	0.45	0.14	0.77	0.10	0.52	0.14
0.75	0.90	0.50	0.66	0.06	0.76	0.05	0.57	0.09	0.60	0.59	0.76	0.49	0.54	0.69
0.75	0.90	0.90	0.73	0.09	0.75	0.09	0.67	0.12	0.72	0.93	0.75	0.90	0.65	1.07
0.95	0.10	0.10	0.95	0.01	0.95	0.01	0.95	0.01	0.91	0.10	0.97	0.10	0.91	0.10
0.95	0.10	0.50	0.95	0.05	0.95	0.05	0.95	0.05	0.93	0.50	0.96	0.50	0.93	0.51
0.95	0.10	0.90	0.95	0.09	0.95	0.09	0.95	0.09	0.95	0.90	0.95	0.90	0.95	0.90
0.95	0.50	0.10	0.94	0.01	0.95	0.01	0.93	0.02	0.72	0.10	0.97	0.10	0.70	0.11
0.95	0.50	0.50	0.94	0.05	0.95	0.05	0.94	0.07	0.82	0.51	0.96	0.50	0.78	0.54
0.95	0.50	0.90	0.95	0.09	0.95	0.09	0.95	0.10	0.93	0.90	0.95	0.90	0.91	0.92
0.95	0.90	0.10	0.74	0.01	0.95	0.01	0.57	0.02	0.55	0.10	0.95	0.10	0.53	0.11
0.95	0.90	0.50	0.84	0.05	0.95	0.05	0.62	0.09	0.73	0.51	0.95	0.50	0.56	0.53
0.95	0.90	0.90	0.93	0.09	0.95	0.09	0.80	0.12	0.91	0.90	0.95	0.90	0.73	0.93
Bounds														
0.50	0.00	0.00	0.50		1.00		0.50							
0.50	1.00	0.00	0.25		0.50		0.50							
1.00	0.00	0.00	1.00		1.00		1.00							
1.00	1.00	0.00	0.50		1.00		0.50							

Table 3: Plim of estimators in the case of omitted dynamics

Structural parameters					Panel A: β=0.1						Panel B: β=1					
					GMM1		GMM2		ML		GMM1		GMM2		ML	
ω_f	ω_b^1	ω_b^2	$\varphi_1+\varphi_2$	ρ	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b
0.40	0.90	-0.30	0.75	0.00	0.31	0.13	0.53	0.05	0.54	0.13	0.31	1.35	0.53	0.53	0.54	1.35
0.40	0.90	-0.30	0.75	0.50	0.32	0.26	0.52	0.07	0.54	0.18	0.36	2.28	0.50	0.86	0.55	1.88
0.40	0.90	-0.30	0.75	0.90	0.42	0.33	0.51	0.20	0.58	0.20	0.42	3.24	0.51	1.97	0.59	2.07
0.60	0.25	0.15	0.92	0.00	0.71	0.09	0.86	0.07	0.53	0.09	0.71	0.88	0.86	0.71	0.53	0.88
0.60	0.25	0.15	0.92	0.50	0.67	0.06	0.81	0.01	0.53	0.08	0.61	0.77	0.67	0.56	0.52	0.78
0.60	0.25	0.15	0.92	0.90	0.54	0.07	0.55	0.06	0.52	0.07	0.54	0.71	0.55	0.62	0.52	0.71
0.60	0.75	-0.35	0.08	0.00	0.44	0.12	0.52	0.11	0.70	0.12	0.44	1.17	0.52	1.09	0.70	1.17
0.60	0.75	-0.35	0.08	0.50	0.46	0.15	0.53	0.14	0.71	0.12	0.50	1.47	0.56	1.40	0.76	1.33
0.60	0.75	-0.35	0.08	0.90	0.53	0.16	0.59	0.16	0.80	0.12	0.55	1.63	0.61	1.60	0.83	1.29
0.80	-0.30	0.50	0.88	0.00	1.60	0.08	2.17	0.06	0.60	0.08	1.60	0.75	2.17	0.57	0.60	0.75
0.80	-0.30	0.50	0.88	0.50	1.42	-0.03	1.99	-0.12	0.58	0.06	0.93	0.38	1.27	-0.12	0.54	0.54
0.80	-0.30	0.50	0.88	0.90	0.61	0.03	0.67	-0.01	0.52	0.04	0.60	0.37	0.63	0.11	0.52	0.44
0.80	0.10	0.10	0.38	0.00	0.89	0.10	0.89	0.10	0.78	0.10	0.89	0.97	0.89	0.97	0.78	0.97
0.80	0.10	0.10	0.38	0.50	0.88	0.09	0.88	0.09	0.77	0.10	0.83	0.91	0.83	0.90	0.74	0.91
0.80	0.10	0.10	0.38	0.90	0.79	0.08	0.79	0.08	0.72	0.09	0.78	0.86	0.78	0.86	0.72	0.86
0.80	0.50	-0.30	-0.13	0.00	0.62	0.11	0.63	0.11	0.85	0.11	0.62	1.06	0.63	1.05	0.85	1.06
0.80	0.50	-0.30	-0.13	0.50	0.63	0.12	0.64	0.12	0.86	0.10	0.68	1.21	0.70	1.21	0.93	1.16
0.80	0.50	-0.30	-0.13	0.90	0.71	0.13	0.73	0.13	0.98	0.11	0.74	1.30	0.76	1.30	1.02	1.21
Bounds																
1.00	1.00	-1.00	-1.00	0.00	0.50		0.50		1.00							
1.00	1.00	-1.00	-1.00	1.00	0.50		0.50		1.00							
1.00	-1.00	1.00	1.00	0.00	infinite		infinite		1.00							
1.00	-1.00	1.00	1.00	1.00	0.50		infinite		0.50							
0.33	1.00	-0.33	1.00	0.00	0.25		0.50		0.50							
0.33	1.00	-0.33	1.00	1.00	0.25		0.50		0.50							

Table 4: Finite-sample properties of estimators in the model with feedback

Structural parameters			Statistic	Panel A: β=0.1								Panel B: β=1							
				GMM (L=0)		GMM (L=7)		ML		Plim OLS		GMM (L=0)		GMM (L=7)		ML		Plim OLS	
ω_f	ρ	γ		ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β	ω_f	β
0.25	0.50	-0.50	median	0.42	0.05	0.48	0.03	0.39	0.06	0.49	0.02	0.25	1.01	0.22	1.05	0.25	1.01	0.15	1.14
			MAD	*0.13*	*0.05*	*0.04*	*0.02*	*0.10*	*0.04*			*0.04*	*0.08*	*0.04*	*0.09*	*0.04*	*0.08*		
0.25	0.75	-0.50	median	0.40	0.05	0.46	0.03	0.34	0.08	0.49	0.02	0.25	1.00	0.23	1.04	0.25	1.00	0.17	1.14
			MAD	*0.11*	*0.04*	*0.04*	*0.02*	*0.09*	*0.03*			*0.03*	*0.07*	*0.03*	*0.08*	*0.03*	*0.07*		
0.25	0.90	-0.50	median	0.33	0.08	0.43	0.04	0.29	0.09	0.48	0.02	0.25	1.01	0.23	1.04	0.25	1.01	0.18	1.13
			MAD	*0.09*	*0.03*	*0.04*	*0.02*	*0.08*	*0.03*			*0.03*	*0.06*	*0.03*	*0.06*	*0.03*	*0.06*		
0.50	0.75	-0.50	median	0.49	0.11	0.47	0.12	0.50	0.11	0.43	0.14	0.49	1.02	0.44	1.07	0.50	1.01	0.29	1.12
			MAD	*0.06*	*0.04*	*0.04*	*0.04*	*0.05*	*0.03*			*0.04*	*0.08*	*0.04*	*0.08*	*0.03*	*0.08*		
0.75	0.75	-0.50	median	0.74	0.11	0.65	0.14	0.75	0.10	0.43	0.23	0.73	1.01	0.64	1.06	0.75	1.00	0.35	1.34
			MAD	*0.06*	*0.04*	*0.06*	*0.04*	*0.05*	*0.03*			*0.06*	*0.09*	*0.05*	*0.10*	*0.04*	*0.09*		
0.25	0.50	-0.10	median	0.47	0.04	0.49	0.03	0.43	0.05	0.50	0.03	0.25	1.00	0.26	1.00	0.25	1.01	0.26	0.99
			MAD	*0.12*	*0.05*	*0.03*	*0.03*	*0.07*	*0.04*			*0.05*	*0.12*	*0.05*	*0.12*	*0.05*	*0.12*		
0.25	0.75	-0.10	median	0.48	0.03	0.49	0.02	0.42	0.05	0.49	0.02	0.25	1.01	0.25	1.02	0.25	1.01	0.24	1.03
			MAD	*0.11*	*0.05*	*0.03*	*0.02*	*0.07*	*0.03*			*0.04*	*0.11*	*0.04*	*0.11*	*0.04*	*0.10*		
0.25	0.90	-0.10	median	0.45	0.03	0.49	0.01	0.38	0.06	0.49	0.01	0.25	1.00	0.25	1.02	0.25	1.00	0.23	1.05
			MAD	*0.11*	*0.04*	*0.03*	*0.02*	*0.07*	*0.03*			*0.03*	*0.08*	*0.03*	*0.08*	*0.03*	*0.08*		
0.50	0.75	-0.10	median	0.50	0.11	0.49	0.11	0.51	0.10	0.48	0.07	0.49	1.02	0.46	1.10	0.50	1.01	0.33	1.35
			MAD	*0.04*	*0.05*	*0.03*	*0.04*	*0.03*	*0.03*			*0.03*	*0.10*	*0.03*	*0.10*	*0.03*	*0.09*		
0.75	0.75	-0.10	median	0.74	0.10	0.67	0.12	0.76	0.10	0.49	0.16	0.73	1.02	0.66	1.07	0.75	1.02	0.44	1.20
			MAD	*0.06*	*0.04*	*0.05*	*0.04*	*0.04*	*0.02*			*0.04*	*0.12*	*0.04*	*0.13*	*0.03*	*0.11*		

Table 5: Plim of estimators in the feedback model with measurement error

Structural parameters				Panel A: β=0.1						Panel B: β=1					
				GMM1		GMM2		ML		GMM1		GMM2		ML	
ω_f	ρ	τ	γ	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b
0.25	0.75	0.10	-0.50	0.47	0.00	0.52	0.00	0.54	0.01	0.44	0.07	0.61	0.04	0.66	0.07
0.25	0.75	0.50	-0.50	0.42	0.02	0.50	0.01	0.46	0.03	0.37	0.40	0.52	0.28	0.52	0.38
0.25	0.75	0.90	-0.50	0.31	0.07	0.43	0.03	0.32	0.07	0.28	0.86	0.34	0.77	0.31	0.84
0.50	0.75	0.50	-0.50	0.54	0.04	0.57	0.04	0.59	0.06	0.56	0.47	0.66	0.42	0.69	0.47
0.75	0.75	0.50	-0.50	0.75	0.05	0.77	0.04	0.79	0.08	0.78	0.50	0.85	0.48	0.86	0.53
0.25	0.50	0.10	-0.50	0.50	0.00	0.53	0.00	0.53	0.00	0.53	0.06	0.68	0.04	0.70	0.07
0.25	0.50	0.50	-0.50	0.49	0.01	0.53	0.00	0.49	0.02	0.44	0.37	0.57	0.27	0.55	0.36
0.25	0.50	0.90	-0.50	0.37	0.06	0.44	0.04	0.36	0.06	0.30	0.84	0.36	0.77	0.32	0.83
0.25	0.90	0.10	-0.50	0.41	0.01	0.51	0.00	0.52	0.01	0.40	0.07	0.57	0.04	0.64	0.08
0.25	0.90	0.50	-0.50	0.37	0.03	0.49	0.01	0.43	0.04	0.35	0.41	0.49	0.28	0.51	0.38
0.25	0.90	0.90	-0.50	0.28	0.08	0.42	0.04	0.30	0.08	0.27	0.86	0.32	0.77	0.30	0.83
0.25	0.75	0.10	-0.10	0.48	0.00	0.53	0.00	0.50	0.01	0.34	0.07	0.58	0.02	0.53	0.08
0.25	0.75	0.50	-0.10	0.43	0.02	0.51	0.01	0.43	0.03	0.31	0.43	0.54	0.13	0.43	0.39
0.25	0.75	0.90	-0.10	0.35	0.06	0.47	0.02	0.34	0.07	0.26	0.88	0.39	0.58	0.29	0.85
0.50	0.75	0.50	-0.10	0.51	0.05	0.55	0.03	0.53	0.08	0.46	0.55	0.61	0.39	0.53	0.61
0.75	0.75	0.50	-0.10	0.73	0.05	0.76	0.05	0.74	0.10	0.66	0.52	0.79	0.48	0.74	0.55
0.25	0.50	0.10	-0.10	0.50	0.00	0.56	0.00	0.50	0.00	0.40	0.07	0.65	0.02	0.52	0.08
0.25	0.50	0.50	-0.10	0.45	0.02	0.55	0.00	0.42	0.03	0.34	0.40	0.61	0.12	0.42	0.39
0.25	0.50	0.90	-0.10	0.35	0.07	0.44	0.04	0.35	0.07	0.28	0.85	0.43	0.58	0.29	0.84
0.25	0.90	0.10	-0.10	0.43	0.00	0.52	0.00	0.49	0.01	0.30	0.08	0.54	0.02	0.52	0.08
0.25	0.90	0.50	-0.10	0.40	0.02	0.51	0.00	0.41	0.04	0.28	0.45	0.51	0.13	0.42	0.39
0.25	0.90	0.90	-0.10	0.30	0.07	0.47	0.02	0.30	0.08	0.26	0.87	0.38	0.56	0.29	0.84

Table 6: Plim of estimators in the feedback model with omitted dynamics

Structural parameters					Panel A: β=0.1						Panel B: β=1					
					GMM1		GMM2		ML		GMM1		GMM2		ML	
ω_f	ω_b^1	ω_b^2	ρ	γ	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b	α_f	b
0.33	0.33	0.34	0.50	-0.50	1.10	-0.09	1.46	-0.18	0.57	0.00	0.76	0.71	0.73	0.73	0.55	0.78
0.33	0.33	0.34	0.75	-0.50	1.06	-0.10	1.32	-0.17	0.56	0.01	0.67	0.60	0.63	0.64	0.51	0.69
0.33	0.33	0.34	0.90	-0.50	0.87	-0.07	1.15	-0.15	0.50	0.02	0.59	0.58	0.56	0.61	0.47	0.66
0.25	0.25	0.50	0.75	-0.50	1.62	-0.17	2.03	-0.25	0.61	0.00	0.88	0.38	0.75	0.49	0.55	0.57
0.50	0.00	0.50	0.75	-0.50	1.59	-0.21	1.89	-0.29	0.71	-0.03	1.18	0.58	0.96	0.71	0.74	0.77
0.50	0.25	0.25	0.75	-0.50	0.88	-0.06	0.97	-0.10	0.58	0.04	0.72	0.79	0.70	0.80	0.59	0.84
0.65	0.35	0.00	0.75	-0.50	0.65	0.10	0.65	0.10	0.65	0.10	0.66	1.00	0.66	1.00	0.65	1.00
0.75	0.75	-0.50	0.75	-0.50	0.46	0.36	0.48	0.34	0.70	0.23	0.41	1.48	0.38	1.51	0.53	1.41
0.75	-0.50	0.75	0.75	-0.50	4.09	-0.51	3.46	-0.38	1.17	0.02	4.74	0.83	1.58	1.51	1.92	1.60
0.33	0.33	0.34	0.50	-0.10	0.94	-0.03	1.59	-0.16	0.47	0.03	0.59	0.52	0.76	0.32	0.42	0.61
0.33	0.33	0.34	0.75	-0.10	1.04	-0.10	1.59	-0.22	0.49	0.02	0.49	0.46	0.58	0.30	0.39	0.54
0.33	0.33	0.34	0.90	-0.10	0.90	-0.09	1.44	-0.23	0.47	0.02	0.42	0.47	0.47	0.37	0.37	0.52
0.25	0.25	0.50	0.75	-0.10	1.46	-0.13	2.11	-0.24	0.52	0.01	0.59	0.24	0.85	-0.10	0.39	0.39
0.50	0.00	0.50	0.75	-0.10	1.47	-0.20	2.24	-0.40	0.53	0.02	0.75	0.31	0.80	0.24	0.49	0.50
0.50	0.25	0.25	0.75	-0.10	0.78	-0.05	1.05	-0.17	0.50	0.04	0.58	0.64	0.60	0.61	0.48	0.69
0.65	0.35	0.00	0.75	-0.10	0.65	0.11	0.65	0.11	0.65	0.10	0.65	1.00	0.65	1.00	0.65	1.00
0.75	0.75	-0.50	0.75	-0.10	0.51	0.20	0.55	0.19	0.83	0.11	0.55	1.53	0.56	1.52	0.81	1.33
0.75	-0.50	0.75	0.75	-0.10	3.19	-0.49	4.07	-0.68	0.68	0.04	1.62	-0.06	1.18	0.27	0.70	0.56

Table 7: GMM and ML estimates of the hybrid Phillips curve with a single lag

	GMM1		GMM2			ML				
						Phillips curve			Output-gap equation	
	Parameter	std dev.	Parameter	std dev.		Parameter	std dev.		Parameter	std dev.
ω_f	0.664	*0.120*	0.697	*0.119*	ω_f	0.451	*0.035*	γ_1	0.055	*0.059*
ω_b	0.326	*0.120*	0.293	*0.119*	ω_b	0.539	*0.035*	γ_2	-0.110	*0.067*
β	-0.027	*0.043*	-0.036	*0.043*	β	0.037	*0.019*	γ_3	0.024	*0.062*
								ρ_1	1.033	*0.077*
								ρ_2	-0.069	*0.111*
								ρ_3	-0.163	*0.076*
	stat.	p-value	stat.	p-value		stat.	p-value		stat.	p-value
J-stat	-		4.645	*0.326*	lnL	-412.82				
					see	0.631		see	0.727	
					$Q(4)$	4.130	*0.042*	$Q(4)$	0.046	*0.831*
					$R(1)$	5.208	*0.023*	$R(1)$	0.432	*0.511*
					J-B	0.443	*0.801*	J-B	2.460	*0.292*

Table 8: ML estimates of the hybrid Phillips curve with three lags

	Phillips curve			Output-gap equation	
	Parameter	std dev.		Parameter	std dev.
ω_f	0.401	*0.143*	γ_1	0.074	*0.055*
ω_b^{1}	0.468	*0.097*	γ_2	-0.101	*0.067*
ω_b^{2}	-0.082	*0.096*	γ_3	-0.010	*0.056*
ω_b^{3}	0.202	*0.069*	ρ_1	1.024	*0.075*
β	0.102	*0.058*	ρ_2	-0.072	*0.110*
			ρ_3	-0.157	*0.076*
	stat.	p-value		stat.	p-value
lnL	-402.74				
see	0.745		see	0.727	
$Q(4)$	0.168	*0.682*	$Q(4)$	0.029	*0.865*
$R(1)$	8.054	*0.005*	$R(1)$	0.503	*0.478*
J-B	2.321	*0.313*	J-B	2.363	*0.307*

Table 9: Finite-sample properties of estimators evaluated by Monte-Carlo simulations

	GMM1		GMM2		ML	
	Median	MAD	Median	MAD	Median	MAD
ω_f	0.559	0.042	0.600	0.052	0.455	0.029
ω_b [1]	0.431	0.042	0.390	0.052	0.535	0.029
β	0.019	0.015	0.004	0.020	0.034	0.011

Figure 1: Domain of validity of the hybrid model with two lags

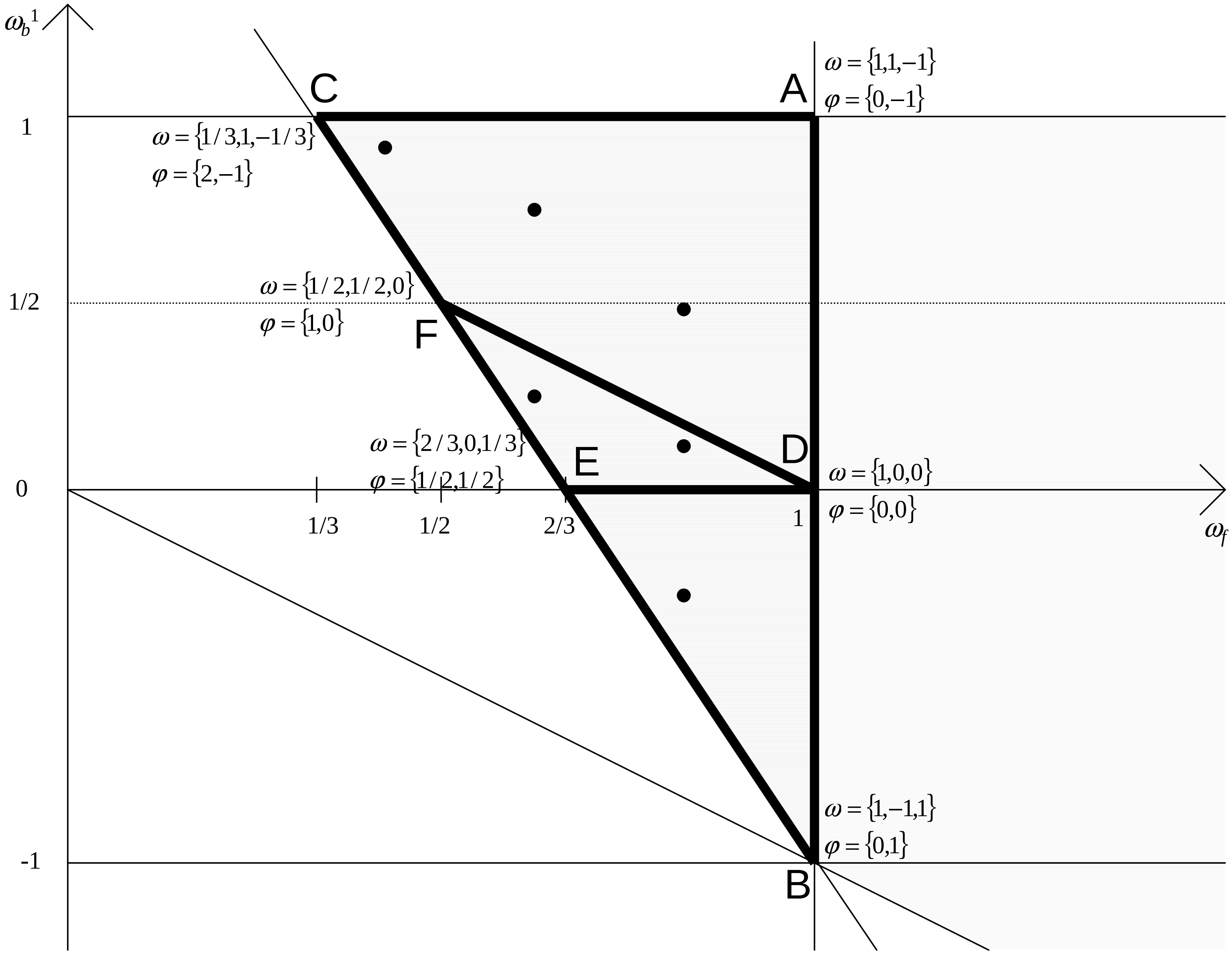
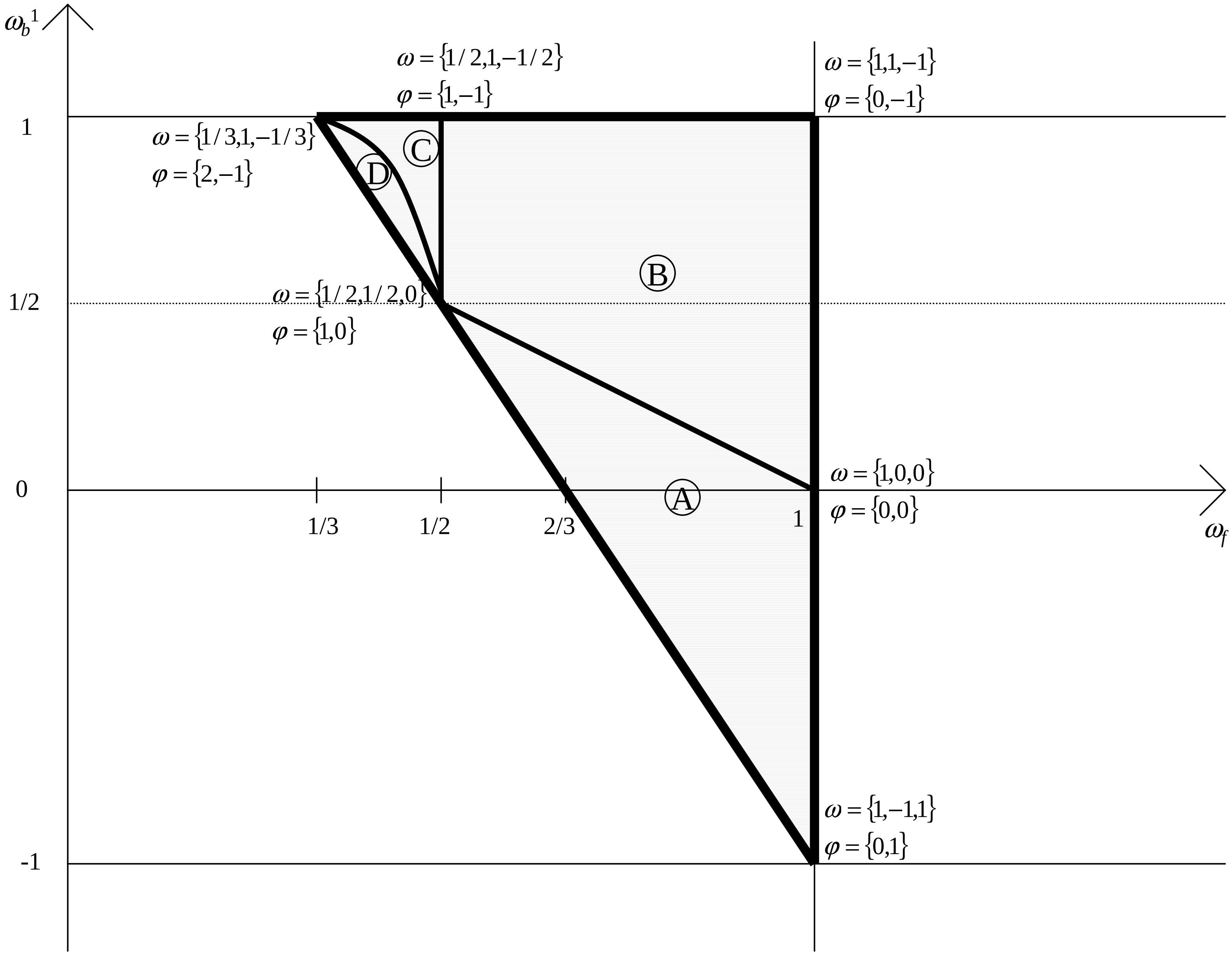


Figure 2: Areas for the different rankings of Plims of estimators in the hybrid model with two lags



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