Are Stablecoins Stable?

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Abstract

Stablecoins are cryptocurrencies designed to peg their value to an official currency. This paper proposes a framework to analyze their stability under the four most relevant pegging mechanisms. The analysis points out to potential fragility in existing stablecoin protocols and policy implications for regulators. Off-ledger currency-backed stablecoins face an increased risk of misappropriation when the demand for the stablecoin falls, leading the stablecoin price to drop below parity. Crypto-collateralized stablecoins are vulnerable to sharp declines in collateral asset value, even when a protective liquidation is triggered early. To admit a stable equilibrium, algorithmic stablecoins require that their demand grows at an exponential rate. Hence, stablecoin price may drop below parity—and potentially to zero—when demand falls. The decentralization of crypto-backed stablecoins improves stability by engineering better incentives for the risk management of individual vaults.

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1 Introduction

A stablecoin is a type of cryptocurrency designed to maintain a stable value vis-à-vis an official currency. Its aim is to solve a fundamental issue of conventional cryptocurrencies such as Bitcoin and Ethereum: These are too volatile to be efficiently used as means of payment or store of value. As such, stablecoins offer to combine the efficiency of the blockchain technology with the stability of well-established currencies. Given the cost of most existing payment systems and the structural demand for safe and inflation-proof assets from developing countries, stablecoins have attracted many investors in the last couple of years. In 2018, the Basis stablecoin project received more than \$100 million in equity investment while Facebook announced its intention to launch its own stablecoin, Libra[1] (since then renamed Diem). Since 2018, the total market capitalization of stablecoins has grown from \$3 billion at the start of 2019 to \$35 billion in January 2021. In spite of the alleged potential of the technology, others have voiced concerns about the actual stability of these stablecoins. In the US, the House of Representatives is currently discussing the STABLE Act.[2] This project would restrict the issuance of stablecoins to chartered institutions. In the UK, the Treasury has launched the "UK regulatory approach to cryptoassets and stablecoins: Consultation and call for evidence".[3]

Despite their relevance in policy debates and connection to well-known theories in economics, these questions have not yet been addressed in the literature. This paper aims to fill this gap by developing a general model of stablecoins to analyze the stability of the various protocols that have flourished in the last couple of years. In the model, a stablecoin protocol consists of a collection of smart-contracts escrowing assets as collateral, setting issuance and redemption rules for the stablecoin, and determining liquidation covenants. Stablecoin protocols share many features with both banks and central banks, varying degrees of similarity across types of stablecoins. For instance, a primary deviation from a bank deposit is that many stablecoins are not directly redeemable outside of liquidation events. We focus on stablecoin protocols with this feature as traditional models of bank runs (Diamond and Dybvig (1983)) would not be directly applicable to them.

Our dynamic model encompasses the four most relevant designs of stablecoin, for which

[1] The project has been progressively and significantly scaled down: the project founders mentioned regulatory setbacks while some commentators pointed to more structural design deficiencies.

[2] Stablecoin Tethering and Bank Licensing Enforcement Act

[3] See HM Treasury (2021)

we derive asset pricing equations in closed form and analyze their stability properties. Additionally, we provide the pricing equations for the protocol's governance tokens. These tokens represent a secondary asset issued by the protocol along with the stablecoin and have similar properties as equity shares in conventional corporations. Governance token holders are entitled to the system's seigniorage revenues and voting rights on the protocol's main parameters in exchange for absorbing the system's residual risk.

First, we consider protocols that are backing the stablecoins fully with currency stored outside the ledger. Following Klages-Mundt et al. (2020), we call these *reserve fund* protocols. In our model, the main vulnerability for this type of protocols originates from the fact that the currency is not contractible on the ledger. When off-ledger institutions are not sufficient to fully prevent some misappropriation of the currency by the governance token holders, a drop in stablecoin demand may lead to the price of the stablecoin falling below parity. This outcome arises because the future seigniorage revenues—or *franchise value*—may then fall to a point at which it is not incentive-compatible not to steal. Moreover, the analysis of reserve fund stablecoins already highlights an important feature of most protocols: governance token investors act as monopolists of the stablecoin and restrict its issuance to maximize seigniorage revenues.

Second, we consider protocols backing the stablecoin through an on-ledger volatile crypto-currency. The pegging mechanism behind this type of protocols relies on locking-in enough collateral and automatically liquidating the system in the instance of a large drop in the collateral asset value. As long as stablecoin holders remain confident that there is enough collateral asset—such that the stablecoin can be liquidated at face value— the stablecoin will trade at parity. We highlight the close connection between the pricing of stablecoins and the one of infinite maturity corporate bonds as in the dynamic corporate finance model of Leland (1994). We solve the model as a recursive equilibrium in its unique state variable: the ratio between the collateral asset value and the outstanding quantity of stablecoins. The solution sheds light on two vulnerabilities for this type of scheme. First, even if the liquidation threshold is set at a collateralization ratio above one—i.e., the protocol liquidates with more collateral asset than stablecoin debt—frictions in the market for the collateral asset may prevent the stablecoin from being liquidated at parity.[4] When stablecoin investors anticipate large enough frictions in collateral asset markets, the price

[4] In practice, the system is likely to liquidate at times when there is heightened volatility in the price of crypto-currency serving as collateral. This feature means that investors seeking stability see their stablecoin investment being swapped into volatile crypto-currency when volatility and downside risks are the highest.

of the stablecoin increasingly falls below parity when the system's collateralization ratio declines.

In addition, it is well known in the corporate finance literature that shareholders tend to resist leverage reductions even when this action increases the value of the firm. Admati, DeMarzo, Hellwig, and Pfleiderer (2018) show that this *leverage ratchet effect* applies to very general settings. This issue is of primary order for crypto-collateralized stablecoins as many of these protocols rely on discretionary reductions in leverage to avoid costly liquidations and preserve the value of the stablecoin. We show that the leverage ratchet effect only partially affects crypto-collateralized schemes thanks to the additional incentives generated by a monopoly position over the stablecoin. Unlike the standard corporate tax shield benefit that is linear in leverage, seigniorage revenues are marginally decreasing because issuing additional stablecoins reduces their scarcity. Hence for some low enough collateralization ratio, it may be beneficial for token holders to reduce the system's leverage. However, as in a standard corporate finance model, reducing leverage also results in the transfer of wealth away from governance token holders to stablecoin investors. When the concavity in the demand function for stablecoin is low, the latter effect can dominate the former for a large part of the state space and the value of the stablecoin may fall significantly below parity before any reduction of leverage is incentives-compatible.

Next, we consider the case of stablecoin protocols that do not use any collateral to back their price but instead rely on continuous adjustments in quantity as their primary pegging mechanism. Examples of these *algorithmic* stablecoins can be found in Nubits and the Basis project before it shut down. This type of protocol differs from the previous ones. Upon issuing and selling new stablecoins, it pays the proceeds from the sale in dividends to the governance token holders. As a consequence, there is no tangible asset left to reimburse stablecoin holders in a liquidation. The protocol resolves this issue by not providing liquidation covenants at all so that the stablecoins are fully irredeemable. For the stablecoin price to be stable, the protocol needs to withdraw stablecoins from the supply when the demand drops. As it does not hold any asset that it can sell to buy back its stablecoins, the protocol creates a new asset pledging future seigniorage revenues. When demand increases again so that new stablecoins have to be minted, the associated seigniorage revenues are allocated to the complete reimbursement of these *seigniorage bonds* before governance tokens holders can receive any new dividend. Our analysis points to two strong vulnerabilities for this type of protocol. The first arises because the demand for the stablecoin depends on its stability, and algorithmic stablecoins always feature a "run" equilibrium with a stablecoin price equal to zero. Secondly, because reducing the supply of stablecoins requires pledging additional future seigniorage revenues, there is a limit to the quantity adjustment mechanism. More precisely, when the discounted value of all future seigniorage revenues has already been pledged, the system is unable to sell new seigniorage bonds so that future adverse shocks in demand result in the stablecoin price dropping below parity. This issue is similar to the one of a central bank with negative equity not being able to control inflation.

Last, we consider the case of decentralized and crypto-collateralized stablecoins that allows for the decentralized creation of new stablecoins by anyone holding enough collateral. Individual investors have the opportunity to lock some collateral assets in a smart contract generated by the protocol and can issue some stablecoins against it. In the process, new stablecoins are created entirely fungible with preexisting stablecoins. Investors unlock their collateral assets by repurchasing and "burning" enough stablecoins to liquidate the position. The system's stability relies on providing the right set of incentives for investors to adopt prudent risk management practices and not over-extend the supply of stablecoins. We find that these incentives are crucial as, otherwise, infinitesimal agents do not internalize the negative externality that leverage has on the system and its franchise value. With the right incentives, we conclude that this approach possesses significant benefits over centralized stablecoins.

Literature review Our paper mainly contributes to the growing literature that addresses the degree of effective stability of stablecoins. Klages-Mundt, Harz, Gudgeon, Liu, and Minca (2020) describe a framework to study the economic risk structure and agents' incentives in different stablecoin systems, incorporating attacks to governance, data feeds as well as deleveraging feedback spirals. From the computer sciences literature, Klages-Mundt and Minca (2019, 2020) develop models featuring endogenous stablecoin price and an exogenous collateral, including deleveraging spirals and liquidation in a system with imperfectly elastic stablecoin demand. Moreover, they formally identify stable and unstable regions for the stablecoin as well as the conditions for the occurrence of a destabilizing deleveraging spiral in the stablecoin market. Gudgeon, Perez, Harz, Livshits, and Gervais (2020) simulate a stress-test scenario for a DeFi protocol and find that, under given parameters, excessive outstanding debt and drying up of liquidity can lead the lending protocol to become quickly undercollateralized. Closely related, Cao, Dai, Kou, Li, and Yang (2018)'s white paper applies option pricing theory to value a stablecoin as a dual-purpose fund contract which normally features low volatility but ultimately requires a stabilizing guarantee during extreme Black Swan events. Finally, our paper relates to the general literature describing the properties of stablecoins, as in Arner, Auer, and Frost (2020); Berentsen and Schär (2019); Bullmann, Klemm, and Pinna (2019); ECB (2019); Eichengreen (2019); G30 (2020).

In studying the stabilization mechanisms across stablecoin types and the failure of governance incentives to recapitalize undercollateralized systems, our paper draws from the corporate finance literature which examines firm shareholders' attitudes towards leverage. Black and Scholes (1973) first documented that, in a frictionless capital structure setting of Modigliani and Miller (1963), firm shareholders do not have incentives to voluntarily buy back debt and reduce leverage as this always implies a transfer of wealth to existing creditors, and they will give up their default option. Leland (1994) attributes this resistance to the reduction in dilution of existing debt since shareholders do not internalize the benefits from lower bankruptcy costs accruing to debtholders but rather pay a higher post-recapitalization price. Closely related, Admati et al. (2018) generalize these findings to multiple asset classes of debt and with agency frictions and document a "leverage ratchet effect", whereby shareholders have no incentives to delever the firm and instead always find it optimal to further increase leverage by issuing new debt, even when leverage exceeds its optimal level. The results are consistent with agency cost models where debt overhang distorts incentives, for instance, through under-investment as in Myers (1977), or asset substitution as in Jensen and Meckling (1976)—where shareholders shift risk towards debtholders by engaging in riskier projects—or under the "control hypothesis" for debt creation in Jensen (1986), in the presence of free cash flow agency costs. DeMarzo and He (2020) also show delevering resistance effects, although in their model leverage mean-reverts to a target because of asset growth and debt maturity. While our paper generally confirms the predictions on the leverage ratchet effect, we extend the model to different stablecoin systems and identify parameters and specific conditions where the unstable leverage incentives dynamics do not hold. As such, we also contribute by studying the extent to which leverage ratcheting effects always create a wedge between the shareholder- and firm-value maximization objective, or, in different words, how different stablecoin systems induce shareholder incentives to be aligned with the goal of total system stability.

More broadly, our paper contributes to the literature applying corporate finance and asset pricing models to model digital platforms and token valuations. While not mainly focusing on stablecoins, Cong, Li, and Wang (2020a) develop a continuous-time model of token-based platform economy with network effects and endogenous token price and also document conflicts of interests between platform owners and users, resulting in an under-investment outcome. In their model as in ours, platform insiders restrict the supply of token to preserve the franchise value, a mechanism stabilizing its price. We highlight the existence of limitations to such a quantity adjustment mechanism. Cong, Li, and Wang (2020b) build a dynamic asset pricing model with network effects and inter-temporal linkages in endogenous token price and user adoption, and analyze the Markov equilibrium with platform productivity as the state variable.

2 A Brief Taxonomy of Stablecoins

According to Bullmann et al. (2019), stablecoins can be formally defined as *"digital units of value that are not a form of any specific currency (or basket thereof) but rely on a set of stabilization tools which are supposed to minimize fluctuations of their price in such currency(ies)"*.

Stablecoins can be categorized into four groups, based on the type of collateral supporting the coin. The simplest type is a *fiat-backed* stablecoin, in which each unit of value, held electronically on a distributed ledger, is pegged one-to-one to the underlying currency (usually the US dollar or another existing numeraire) and can always be redeemed for the same amount in the currency of denomination. While presenting the lowest degree of innovation, these initiatives account for the greatest proportion of current stablecoin systems, with the largest and most popular stablecoins by market capitalization being, by order, Tether, USDCoin, Binance USD, Paxos Standard, HUSD and TrueUSD.[5]

The second type is an *off-chain collateralized* stablecoin, in which the coin in a given reference currency is backed by one or multiple traditional assets that are not stored digitally (e.g. commodities or real estate) and whose price in the reference currency fluctuates over time, and hence they will usually require some degree of overcollateralization.

The third type, *on-chain collateralized* stablecoins are backed by a (multiple) crypto-

[5] As of 1/28/2021 (source: https://www.coingecko.com/en/stablecoins)

asset(s) which are stored digitally, with the most popular being Dai, collateralized by the Ether cryptocurrency.

Finally, *algorithmic* stablecoins are tokens which are not explicitly backed by any collateral and rely on dynamic adjustments in the supply of the stablecoin to maintain the peg relative to the currency of reference, as codified in the stablecoin digital protocols. Examples of such stablecoins are NuBits, bitUSD, Terra or Steem Dollars.

Fiat-backed and off-chain collateralized stablecoins represent *custodial* systems as they rely on entrusted depository entities to hold the reserve asset off-chain, against which they issue the digital token. Such systems can be fully collateralized, in which case they most closely resemble traditional narrow banking models where deposits at a commercial bank are fully backed by the underlying reserves. Alternatively, in partially-collateralised systems, the reserve assets represent only a fraction of the total stablecoin supply, and custodial funds also include capital assets which earn interest and support the remaining token supply (e.g. being liquidated to manage excessive stablecoin redemptions), as in traditional fractional reserve banking models. Holders of the stablecoins then have a claim against the custodial assets and are exposed to both counterparty and bank-run risks in the custodian entity.

Conversely, on-chain collateralized and algorithmic stablecoins are *non-custodial* systems which purposely avoid counterparty risks as they rely on smart contracts and a blockchain technology to hold the asset on-chain and define the economic structure between participants. These are decentralized systems which transfer risk from the stablecoin holders to speculative agents who determine stablecoin issuance and absorb risk and profit by holding a leveraged position in the collateral asset. These systems also rely on an "oracle" to disclose the target stablecoin price, and involve a dynamic liquidation and deleveraging process to reduce stablecoin supply whenever the value of collateral falls below a given collateralization threshold, or if stablecoin holders can redeem stablecoins for the collateral. Additionally, they rely on a governance mechanism, represented by voting from governance token holders or a set of algorithms, which defines the parameters governing the functioning of the entire system. As such, decentralized systems are exposed to different types of risks, including deleveraging spirals, data feeds, mining risks, and oracle or governance manipulation.

Following Klages-Mundt et al. (2020), non-custodial stablecoins can be further categorized based on the nature of the primary value in the system, which represents the structure of the collateral as base value for the stablecoin.

An *exogenous* collateral has primary external uses beyond the stablecoin system (e.g. Ether in the Maker system), while an *endogenous* collateral is an asset which is created precisely with the aim of serving as collateral for the stablecoin (e.g. seigniorage shares bitShares in bitUSD). In the latter system, the value of the collateral and the stablecoin are tightly connected through endogenous feedback effects on valuation expectations. Therefore, the stability of either will rely on coordinated confidence in the entire stablecoin system. For example, in the Steemit platform, the collateral asset, Steem, is only partially exogenous as its external primary use is to serve as collateral against which the governance token, Steem Power, is redeemed, although it also supports the stablecoin Steem Dollars, which can be redeemed against this collateral. Finally, if stablecoins do not have an explicit tokenized collateral, they can rely on an *implicit* collateral and depend on market forces to influence speculators' incentives and stabilize the system. For example, in NuBits, in the event of low stablecoin demand, the stabilization mechanism relies on the incentives of risk absorbers to temporarily reduce their supply of stablecoins from the system ("parking") in exchange for future rewards, in expectation of greater demand in the stablecoin which will restore equilibrium and future supply increases (i.e., new shares are awarded). As in endogenous collateral systems, price stabilization in implicit collateral systems, therefore, depends on self-fulfilling expectations and overall confidence in the system.

[Figure 1 about here]

Figure 1 illustrates the price dynamics for four different types of stablecoins, which all suffered crashes and de-pegging events at different times. The top-left panel illustrates the price of USD Tether. While fiat-based stablecoins are supposedly highly secure as each token is 100% backed, Tether experienced a mild de-peg in October 2018 following public concerns that the company behind Tether did not have sufficient funds to back the entire stablecoin supply (Suberg, 2019), in line with typical bank-run risks.

The top-right panel illustrates the deleveraging spiral and volatility increase suffered by Dai on March 12, 2020, popularly denoted as "Black Thursday", amidst the COVID-19 pandemic market crisis, and with crypto-markets plunging more than 50% in value. On that day, high volumes on the Ethereum blockchain caused network congestion and drastic gas price (i.e., transaction fees) increases, leading to oracle delays (i.e., delayed price updates) and cascading collateral liquidations, with several auctions won by zero-bidders

(MakerDAO, 2020; Topbottom, 2020). In turn, with insufficient collateral liquidation revenues and Dai buffer, the equity token (Maker) holders were forced to recapitalize by issuing new Makers and dilute their holdings, as illustrated by the shaded area. Additionally, high demand for Dai during the crypto-market crisis caused the stablecoin to trade at a high premium, as shown by the solid line. This was further exacerbated by buying pressure of speculators who needed to delever their positions, in turn contributing to the drying up of Dai liquidity and further entangling the ability of speculators to delever. Hence, despite the exogenous nature of the collateral, the deleveraging events showed the possibility of feedback effects between the stablecoin and collateral asset.

The bottom-left panel illustrates the two depeggging events of NuBits in May 26, 2016 and March 21, 2018 following a loss of confidence in the stablecoin. The first peg broke due to substitution effects from the crypto-market, following the spike in Bitcoin price which spurred selling in the stablecoin. The coin then broke its peg permanently in March 2018, following insufficient equity reserves in the midst of a demand shock (arguably due to the drop in Bitcoin price in which the reserves were denominated; Klages-Mundt, 2018). Finally, the bottom-right panel illustrates the price history for bitUSD, which broke its peg in December 2018, when it experienced a global settlement due to under-collateralized debt positions (triggered by the drop in the collateral ratio below the maximum short-squeeze ratio allowed by the system; Liu, 2018).

On the basis of such empirical evidence of stablecoin vulnerabilities and considerable heterogeneity in underlying mechanisms, we develop in the next section a general model to jointly study the price dynamics of the tokens in the stablecoin system.

3 A General Model of Stablecoins

We first consider a general model of centralized stablecoins. A stablecoin protocol is a collection of perfectly enforceable on-chain smart contracts to create the stablecoin and a governance token against collateral. The protocol determines (i) stablecoin issuance and withdrawal rules, (ii) liquidation covenants, and (iii) distribution rules for claims at liquidation and dividend payments. At origination, the protocol locks in a smart contract a certain amount of collateral asset—i.e., volatile cryptocurrencies and/or currency buffer—and issues stablecoins against it. The system can be understood with a banking analogy: the volatile collateral assets are akin to loans and securities, the currency buffer is akin to reserves, and the stablecoin is akin to a bank deposit. A critical distinction between bank deposits and stablecoins is that the latter are often not directly redeemable for currency or collateral, so that the pegging mechanism relies on either managing expectations during liquidation or on algorithmic adjustments in quantity.

We capture these elements by adapting standard dynamic corporate finance models with infinite maturity à la Leland (1994). In our model, the main trade-off for the system's leverage choice is between increasing liquidation costs and increasing seigniorage revenues. Liquidation costs occur as a result of liquidity frictions in the secondary market for the collateral asset. Seigniorage revenues are created by the protocol acting as a monopolist of its own stablecoin and restricting supply to maintain a positive spread between the discount rate and the interest accruing on the stablecoin. Thanks to the convenience yields, investors are willing to hold stablecoins paying an interest rate below the discount factor. This trade-off differs from the standard theory of balancing bankruptcy costs versus tax shield benefits in that seigniorage revenues can reach a threshold at which they have negative returns to scale with respect to leverage. We demonstrate below that this feature has important implications for the stability of stablecoins.

3.1 Collateral Asset, Stablecoin, and Liquidation

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions. Time is continuous with $t \in [0, \infty)$. Consider a crypto-asset available on the ledger with value A_t that follows an exogenous diffusion process with constant volatility:

$$\frac{dA_t}{A_t} = \mu(A_t, t)dt + \sigma dZ_t,$$

where dZ_t is the increment of a standard Brownian motion adapted to $\mathcal{F}$. Denote $A_t^\ell \leq A_t$ the value of crypto-assets locked into smart contracts as collateral for the stablecoin protocol. Denote by C_t the aggregate quantity of stablecoins issued by the protocol that pay a constant positive interest rate $r^c > 0$ and trade in secondary markets at price p_t. Stablecoins have an infinite maturity and remain outstanding until they are either repurchased or liquidated by the system. As in Leland (1994), we assume that the process for A_t is unaffected by the leverage of the protocol. We also assume the existence of a riskless rate r that is only accessible through an off-ledger storage technology indexed to the unit of account. Only *on-ledger* assets can be used to write auto-enforceable smart contracts whereas *off-ledger* assets have to be incentive-compatible. More precisely, when the buffer is held off-ledger, the governance token holders can steal a share $1 - \gamma$ of the buffer. The parameter γ can be interpreted as representing the strength of the institutions existing off-ledger. A protocol can incorporate a buffer of off-ledger reserves with face value B_t paying an interest r and a buffer of on-ledger reserves with face value B_t^ℓ paying an interest r^c, the prevailing rate for on-ledger stablecoins.

The first purpose of the protocol is to maintain the price of the stablecoin p_t at parity with respect to the unit of account. In doing so, the protocol is able to collect seigniorage revenues by acting as a monopolist and capturing part of the transaction services it provides to holders of the stablecoin. We capture this stability benefit—or *convenience yield*—of stablecoins by writing the marginal utility of holding one more unit of stablecoin per unit of wealth A_t for a representative investor as:

$$u'(C_t/A_t) = \begin{cases} \max\left\{0, \lambda_1 - \lambda_2\lambda_1 C_t/A_t\right\}, & \text{if } \mid p-1 \mid < \lambda_3 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

In the previous equation, $\lambda_1 >$ and $\lambda_2 \geq 0$ are such that $u(C/A)$ is decreasing and weakly concave and $\lambda_3 \leq 1$ rules how much deviation from parity are tolerated before the convenience yield falls to zero. The maximum operator guarantees that the marginal utility of holding a stablecoin stays positive. All seigniorage revenues accrue to the holder of a secondary asset created by the protocol: governance tokens with value G_t. Holders of these governance tokens act as the decentralized management of the system and decide on the two key parameters of the system: the constant interest rate that is paid on the stablecoin r^c and a stablecoin issuance rule dC_t. These tokens are the effective equity of the system and play a key role in its stability.

To protect the value of stablecoins and the system's franchise value, the protocol may include a smart contract triggering the liquidation of the system when the ratio of collateral assets to stablecoins $\eta_t = A_t/C_t$ falls to a threshold $\underline{\eta}$. When the threshold is reached, collateral assets are liquidated at a discount α reflecting fire-sale frictions in the secondary market for the collateral asset. Stablecoin holders have the seniority on the proceeds up to their face value. Governance token holders receive the residual of the proceeds after stablecoin holders have been paid, if any. If the proceeds fall short of the quantity of stablecoins valued at parity, stablecoin owners are entitled to a share of the proceeds proportional to their holdings. This liquidation process is similar to a firm with limited liability repaying its debtors upon default.

In the next sections, we discuss our assumptions and specialize the model based on existing examples of stablecoin protocols and highlight their respective specificity and vulnerability. All proofs are relegated to Appendix A.

3.2 Discussion of the Environment

Exogenous collateral asset value We take the evolution of the value of the collateral asset A_t as exogenous to clarify that our results do not depend on a particular modeling choice for volatile crypto-currencies. We refer to the work of Cong, Li, and Wang (2020b) for an alternative approach. We also note a movement towards the "tokenization" of more conventional dividend-yielding financial assets that will become usable as additional collateral for stablecoins.

Riskless rates We assume that the off-ledger riskless rate is equal to the discount factor while the on-ledger riskless rate is equal to the stablecoin rate because these two rates are the only ones consistent with a no-arbitrage condition, both on- and off-ledger.

Demand for stablecoins We model the demand for stablecoins as a concave function as in the money-in-the-utility literature dating back to Sidrauski (1967). This approach could easily be substituted by a micro-founded demand for a stable means of exchange minimizing an exchange rate risk while shopping (Williamson and Wright, 2010). Alternatively, one could also imagine to derive the value of the stablecoin as a store of value in an economy with a currency prone to hyper-inflation.

4 Off-ledger Reserve Funds

[Figure 2 about here]

We first consider the simplest case of a stablecoin protocol consisting of the system holding a large enough off-ledger currency buffer to cover all stablecoins at parity. This *narrow bank* case, although the least innovative conceptually, accounts for the largest market share of existing stablecoins, with Tether as a prime example. The benefit of this type of stablecoin is to not rely on the value of a volatile asset as collateral, whereas its vulnerability lies in the currency buffer being stored outside the ledger and therefore not being protected by a smart contract. Figure 2 proposes a balance sheet representation for this type of protocol. Stablecoins are fully backed by the off-ledger currency buffer so that the market value of governance tokens exactly coincides with the system's franchise value reflecting discounted future seigniorage revenues.

4.1 Pricing Equations

Because the value of the collateral asset is not relevant for this type of stablecoin, we set $\mu_t = 0$, $\sigma_t = 0$ and normalize $A_t^\ell = 1$. When this is the case, the model has a static solution, which provides economic intuition about the system's franchise value. This intuition remains valid when analyzing more complex protocols below. The solution for the price of the stablecoin is:

$$p = \frac{r^c + u'(C)}{r},$$

whereas the valuation for the governance token is given by:

$$G = B - \frac{r^c C}{r},$$

so that the total value of the centralized system is given by:

$$S = G + pC = B + \frac{u'(C)C}{r}.$$

The second term of this equation captures the system's franchise value created by the wedge between the stablecoin external valuation from households—taking into consideration the utility flow generated by holding the stablecoin ($r^c + u'(C)$)—and the internal valuation as debt paying an interest rate r^c.

4.2 Pegging Mechanism

In this riskless setting, it is trivial that ensuring the parity (i.e., $p = 1$) requires that

$$r^c = r - u'(C). \tag{2}$$

The previous equation has a straightforward interpretation. For the price of the stablecoin to equal one, its overall return—including the convenience yield $u'(C)$—must be exactly equal to the discount factor r. As governance token holders control both parameters r^c and C at the origination of the system, the previous equation is underdetermined,[6] so that the degree of freedom can be used to maximize the franchise value $F = (r - r^c)C/r$.

4.3 Stability

Applying the functional form from equation (1), we can derive the demand for C from equation (2) as:

$$\frac{C}{A} = \frac{\lambda_1 - (r - r^c)}{\lambda_1 \lambda_2}.$$

The maximization problem for the franchise value is then:

$$\max_{(r-r^c)} \left\{ \frac{\lambda_1 (r - r^c) - (r - r^c)^2}{\lambda_1 \lambda_2} \right\}.$$

Taking the first-order conditions with respect to the spread $r - r^c$, the governance variables $\{r^c, C\}$ that jointly maintain the parity and maximize the franchise value are given by:

$$\{r^{c\star}, C^\star\} = \left\{ r - \frac{\lambda_1}{2}, \frac{1}{2\lambda_2} \right\}. \tag{3}$$

Note that the optimal quantity of stablecoin is decreasing in the concavity of the convenience yield. The intuition is that the governance token holders act as monopolists of their stablecoins and restrict the supply to prevent the marginal utility of stablecoins—and hence the stablecoin spread—from falling to zero. At first, it appears that welfare would be maximized with a large enough supply of stablecoins such that the spread is nil. Proposition 1 contradicts this intuition, pointing that incentive compatibility requires a large enough franchise value for the system's (governance token) owners not to run with the off-ledger currency buffer.

Proposition 1 (Off-Ledger Reserve Fund Vulnerability). *Under the restrictions that*

[6] A similar underdetermination arises for central banks in the implementation of monetary policy as a central bank chooses both the interest paid on its reserves and the quantity of reserves available to banks. The central bank can use the extra degree of freedom for other purposes such as targeting the equilibrium level of liquidity risk in banks (Vandeweyer, 2019).

$\mu_t = 0$, $\sigma_t = 0$, and the normalization $A = 1$, a stablecoin protocol holding an off-ledger buffer that exactly matches the parity value of stablecoins $B = C$ exists and is stable if and only if the franchise value is above the stealable portion of the buffer:

$$\frac{u'(r)C}{r} > (1 - \gamma)B.$$

Equivalently, when taking into account the functional form given in equation (1) and the optimal couple in equation (3):

$$\frac{\lambda_1}{2\lambda_2} > (1 - \gamma).$$

Proof: The proposition is trivially derived from the observation that there is no stochastic movement and, hence, no price deviation from parity as long no-stealing is incentives compatible. □

Proposition 1 establishes that the stability of a stablecoin issued by the reserve fund protocol is vulnerable to a sudden decrease in the marginal convenience yield for the stablecoin. Such an event could be the consequence of a decrease in the preference parameter λ_1 relative to λ_2. In an alternative model, it could also be due to the entry of a competitor with a close substitute stablecoin, such as another private stablecoin protocol or a Central Bank Digital Currency (CBDC). The larger the parameter γ—reflecting better off-ledger institutions—the more an off-ledger reserve fund is stable.

5 Crypto-Collateralized Protocols

[Figure 3 about here]

In this section, we develop the pricing equations for stablecoins and governance tokens issued by a centralized crypto-collateralized protocol, and analyze the stability of the protocol. We highlight the role of liquidity frictions in the secondary market for the collateral asset and potential resistance to leverage reduction—a milder version of leverage ratchet effect of Admati et al. (2018)—as key vulnerabilities of this category of protocols. Figure 3 provides a balance sheet representation for this type of protocol. The system backs the issuance of stablecoins with more volatile collateral assets than stablecoins valued at parity. This overcollateralization creates a book equity buffer in case the collateral asset suddenly looses its value. The market value of governance tokens (the system's market equity) also includes discounted future seigniorage revenues constituting the system's franchise value.

5.1 Markov Equilibrium

In a crypto-collateralized protocol, the stability of stablecoin price depends on the evolution of the value of the collateral asset and the financial structure of the system at each point in time. Without loss of generality, we assume that every collateral asset outstanding is locked in the stablecoin protocol at its origination so that $A_t^\ell = A_t$. Thanks to the scale invariance of our model, we economize on one state variable and focus on Markov perfect equilibria in which the unique relevant state variable is the collateralization ratio $\eta_t = A_t/C_t$. We do so by dividing each previously defined quantity variable by C_t and denote it by a corresponding lower-case notation: $b = B_t/C_t$ and $g = G_t/C_t$. Applying Ito's lemma we find the law of motion for η_t as:

$$d\eta_t = \mu\eta_t dt + \sigma dZ_t \tag{4}$$

Recall that the HJB of an asset $V(\eta)$ paying an interest r^v with constant drift[7] $\mu(A_t, t) = \mu$ is given by:[8]

$$rV(\eta) = r^v + \mu\eta V'(\eta) + (1/2)\sigma^2\eta^2 V''(\eta), \tag{5}$$

with general solution:

$$V(\eta) = \frac{r^v}{r} + c_1\eta^{-\nu} + c_2\eta^{-\overline{\nu}}, \tag{6}$$

and where the two roots λ and $\overline{\lambda}$ are given by:

$$\nu = \frac{2\mu - \sigma^2 + 2\sqrt{2r\sigma^2 + (\sigma^2/2 - \mu)^2}}{2\sigma^2} > 0, \quad \overline{\nu} = \frac{2\mu - \sigma^2 - 2\sqrt{2r\sigma^2 + (\sigma^2/2 - \mu)^2}}{2\sigma^2} < 0.$$

[7]Note that we could consider a more general drift and consider the diffusion equation under the risk-neutral probability measure. In this case, as is common in the mathematical finance literature, by arbitrage, the drift would become r (instead of μ). For clarity of exposition, we abstract from these weighting techniques via the risk-neutral measure and consider a constant μ.

[8]See Black and Cox (1976) and Leland (1994)

The constants c_1 and c_2 are determined by the boundary conditions for the different securities.[9]

5.2 Pricing Equations

Using this general pricing formula, we can determine the market value of the securities such as the stablecoin price $p(\eta)$ and the value of the governance token $g(\eta)$ – i.e., the system equity. This will depend on the benefit received at liquidation: Therefore, the boundaries of the stablecoin price are given by:

$$p(\underline{\eta}) = \min\left\{1, (1-\alpha)\underline{\eta}\right\}, \quad \text{and} \quad \lim_{\eta\to\infty} p(\eta) = \frac{r^c}{r}.$$

Solving for c_1 and c_2 in the general solution (6), one finds the pricing equation for the stablecoin:

$$p(\eta) = \frac{r^c + u'(\eta)}{r}\left(1 - \left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}\right) + \min\left\{1, (1-\alpha)\underline{\eta}\right\}\left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}$$

This equation is comparable to the solution of Leland (1994). The stablecoin promises a perpetual payment of the interest rate r^c and the flow of utility $u'(\eta)$, unless the system liquidates. At liquidation, if there is enough collateral, the stablecoin owners receive the face value of their debt. Otherwise, stablecoin owners receive a share of the proceeds proportional to their holdings. In that equation, the term $(\eta/\underline{\eta})^{-\nu}$ has the interpretation of the expected present value of \$1 contingent on future liquidation of the system.

Similarly, as governance token holders act as the residual claimants (i.e., equity holders), the price of the governance token has the following limit properties:

$$g(\underline{\eta}) = \max\left\{0, (1-\alpha)\underline{\eta} - 1\right\} \quad \text{and} \quad \lim_{\eta\to\infty} g(\eta) = \eta - \frac{r^c}{r}.$$

Thus, the value of the governance token per unit of stablecoin is then given by:

$$g(\eta) = \left(\eta - \frac{r^c}{r}\right)\left(1 - \left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}\right) + \max\left\{0, (1-\alpha)\underline{\eta} - 1\right\}\left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}$$

[9] The securities we consider have upper boundaries that are linear in η and therefore we can rule out the explosive part of the solution by setting $c_2 = 0$.

The value of the governance token is equal to the value of the collateral minus the cost of the payments to the stablecoin owners, adjusted for the liquidation of the system. At the liquidation of the system, if the fire-sale friction on the secondary market is positive $\alpha > 0$ the price of the token is non-zero.

5.3 Pegging Mechanism

Essentially, crypto-collateralized protocols derive their stability from guaranteeing that enough collateral is locked in to protect the value of the stablecoin at liquidation. For this reason, the interest rate paid on the stablecoin has to be consistent with the stablecoin price trading at parity when $\eta \to \infty$. As for reserve funds, this condition is ensured when

$$r^c(\eta) = r - u'(\eta).$$

Note that assuming such a rule for the interest on stablecoins restricts the ability of the protocol to maintain the parity. In particular, setting-up a dynamic rule for $r^c(\eta)$ that exactly compensates coin holders for liquidation risk would effectively maintain the parity for any value of η. In practice, it may be difficult to implement such a rule. For instance, MakerDAO—the decentralized organization behind the DAI stablecoin—has opted to separate *"monetary policy"* considerations involving changes in the system's interest rates in reaction to demand shocks from *"risk management"* considerations.

5.4 Stability

The two pricing equations are analogous to, respectively, the equations for the pricing of (infinite maturity) corporate debt and corporate equity in standard corporate finance dynamic models (Leland, 1994). The main divergence arises because the motive for leverage in our setting originates from the convenience yield of debt (stablecoin) rather than an increase in tax protection. It is well established in the corporate finance literature that once debt has been funded, shareholders tend to resist any form of leverage reduction irrespective of how much it would increase the total value of the firm. Admati et al. (2018) and DeMarzo and He (2020) demonstrate that this leverage ratchet effect applies perfectly to a wide variety of corporate finance models with the standard bankruptcy cost versus tax shield trade-off, so that shareholders never take any action to reduce the firm's leverage.

Such a result is of prime importance in the analysis of the stability of stablecoins as many of the existing protocols rely on active leverage reduction, or equivalently equity injection, to reduce liquidation risks and to stabilize the stablecoin price after a large negative shock to the value of the collateral.

Proposition 2 shows that the leverage ratchet effect of Admati et al. (2018) carries to our stablecoin setting, but potentially only in a subset of the state space, depending on the concavity of the convenience yield. The intuition behind this result is that unlike a standard tax shield that is linear in the quantity of debt issued, the convenience yield is marginally decreasing in the quantity of stablecoin outstanding per unit of wealth. That is, the economy can be saturated in the quantity of stablecoins such that when the leverage is sufficiently high, the convenience yield and the franchise value fall to zero. Hence, by actively reducing leverage, governance token holders appropriate some of the increase in franchise value brought upon by the reduction in leverage.

Proposition 2 (Ratchet effect in centralized crypto-collateralized protocols). *Assume that investors fully tolerate stablecoin volatility, $\lambda_3 = 1$, and that post-liquidation collateral is below stablecoin face value, $(1-\alpha)\underline{\eta} < 1$. Upon receiving the opportunity, governance token holders will approve a reduction in the system's leverage from $1/\eta$ to $1/\eta^\star$ with $\eta^* > \eta$ if and only if*

$$\frac{\lambda_1\lambda_2(1/\eta - 1/\eta^\star) - \lambda_1}{r} > (1-\alpha)\underline{\eta}\left(\frac{\eta^\star}{\underline{\eta}}\right)^{-\nu}$$

Proof: See Appendix A.

[Figure 4 about here]

Figure 4 depicts the dynamics for the price of the stablecoin and illustrates the result from Proposition 2. When the residual value of collateral after liquidation is below parity, $(1-\alpha)\underline{\eta} < 1$, the price of stablecoins is a concave function of the overcollateralization ratio η. It converges to parity on the right and falls to liquidation value $(1-\alpha)\underline{\eta}$ at the liquidation threshold $\underline{\eta}$. In the two panels, $\eta = \eta_r$ denotes the threshold below which it becomes profitable for the token holders to reduce the leverage of the system (or increase its collateralization ratio). Hence, we label the region defined between η_r and $\eta^\star$ as the *inaction region*. In a model in which governance is able to adjust the collateralization ratio at will, the dynamics within this inaction region are unaffected whereas η_r and $\eta^\star$ are transformed into reflecting boundaries for η.

Figure 4 also illustrates how the size of this inaction region—depending on the concavity of the convenience yields—impacts the stability of the stablecoin. With a small inaction region as in Panel (a), only small deviations from parity appear in equilibrium and price volatility is minimal. With a large inaction region, as depicted in Panel (b), large drops from parity are possible as a result of large negative swings in the collateral asset value. At the extreme, when there is little concavity in the convenience yield, the inaction region can span the whole interval between the liquidation threshold $\underline{\eta}$ and the optimal collateralization ratio $\eta^\star$, so that the leverage ratchet effect of Admati et al. (2018) takes over.

6 Uncollateralized Protocols

[Figure 5 about here]

In this section, we apply our model to the analysis of algorithmic stablecoin protocols such as Nubits or Basis. As explained earlier, algorithmic protocols cannot rely on the value of a collateral asset at liquidation as pegging mechanism. Rather, they depend on smart contracts automatically adjusting the supply of stablecoins to the demand. We show here that this strategy has some benefits, but also important limitations.

The structure of an algorithmic protocol can be visualized through the two balance sheets depicted in Figure 5. The balance sheet on the left is the book representation of the system when taking into account only tangible assets and liabilities: the stablecoin is valued as a liability and the franchise value is not accounted for. Considering the system does not feature any tangible collateral asset, the system appears with a negative book equity—i.e, a negative value for the governance token. In contrast, the market balance sheet on the left only takes into account assets and liabilities that are relevant for investors. This accounting includes the franchise value arising from future dividends to governance token holders and does not include stablecoins, which in the absence of a liquidation mechanism are fully irredeemable.

[Figure 6 about here]

The market value of governance tokens is therefore equal to the system franchise value minus the part of this franchise value that has been pledged in seigniorage bonds. As discussed in Section 2, these seigniorage bonds are important to reduce the supply of stablecoins in response to a negative demand shock. These adjustments are illustrated in Figure 6. In the top balance sheets, the system reacts to an increase in the demand for stablecoins by printing more and immediately paying the newly minted stablecoins in dividend to governance token holders. In the bottom balance sheets, the system reacts to a drop in the demand for stablecoins by swapping some stablecoins for seigniorage bonds in a transaction that is similar to an open market operation from a central bank. Selling these seigniorage bonds effectively decreases the stream of seigniorage that will be paid as dividends to governance token holders in the future and, hence, diminishes the market value of the tokens.

6.1 Pricing Equations and Pegging Mechanism

Formally, we specialize our model by setting $A_t^\ell = A_t = B^\ell = B_t = 0, \forall t$ and $\underline{\eta} < 0$, so that the the system never liquidates. Furthermore, since an algorithmic protocol relies on quantity adjustments as pegging mechanism, we assume that it does not pay any interest on the stablecoin.[10] Hence, conditionally on being locally stable ($\sigma_t^p = 0$) the pricing equation for the stablecoin is:

$$p_t = \frac{u'(C_t/A_t)}{r}.$$

To maintain the parity, the protocol must dynamically adjust stablecoin quantity so that the previous equation holds at any point in time. Applying our functional form for $u'(C_t/A_t)$, this condition can be written as:

$$C_t = \frac{\lambda_1 - r}{\lambda_2 \lambda_1} A_t.$$

Applying Ito's lemma, the stabilizing process for C_t follows:

$$\frac{dC_t}{C_t} = \mu dt + \sigma dZ_t. \tag{7}$$

As the demand for stablecoins is proportional to the value of the crypto-asset, the protocol needs to adjust the quantity of stablecoins in line with the process for A_t, i.e., the protocol

[10] Our results do not depend on this assumption, but it clarifies the exposition.

reacts to a negative realisation of the Brownian motion by reducing the supply of stablecoins by a similar proportion. To allow for the repurchase of stablecoins following a negative shock, the system mints new claims to future seigniorage revenues. These seigniorage bonds have the seniority over governance tokens in receiving future seigniorage revenues. To capture this feature, we define $\theta_t \in [0, +\infty]$ as the share of the franchise value that is pledged to bond holders. When this share reaches one, the system is unable to pledge additional future seigniorage to reduce the supply of stablecoin. At this point, the θ variable changes its interpretation and tracks the excess supply of stablecoins. Applying Ito's lemma again, we find the process for θ_t:

$$\frac{d\theta_t}{\theta_t} = -\mu dt - \sigma dZ_t,$$

with an absorbing boundary condition at $\theta_t = 0$. We write the dividend process for the governance token as:

$$dv_t = \begin{cases} \mu dt + \sigma dZ_t, & \text{if } \theta_t = 0, \\ 0, & \text{otherwise.} \end{cases}$$

When θ_t is above zero, all seigniorage revenues generated from issuing additional stablecoins are used to repay the bonds. As new stablecoins are issued, θ_t converges to zero and the protocol starts paying dividends to governance token holders again. The pricing equation for the governance token is therefore:

$$g(\theta) = \frac{(1-\theta)\mu}{r}.$$

We can then write the best feasible quantity adjustment rule as the stopped diffusion process:

$$\frac{dC_t}{C_t} = \begin{cases} \mu dt + \sigma dZ_t, & \text{if } \theta_t \leq 1, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

6.2 Stability

Uncollateralized stablecoins suffer from two weaknesses. First, even if adjustments to the supply of stablecoins provide some local stability, these are not sufficient to ensure the global stability of the protocol and rule out $p_t = 0$ as an equilibrium. Note that this result differs from the cases of reserve-backed and crypto-collateralizated stablecoins. In those cases, the residual value at liquidation pins down the value of the stablecoin at strictly positive levels.

Proposition 3 (Existence of a Run Equilibrium in Algorithmic Protocols). *Assume some intolerance for deviation from parity ($\lambda_3 > 0$. Then, any algorithmic protocol—that is, $A^\ell(\theta) = B(\theta) = B^\ell(\theta) = 0\,\forall\theta$—that adjusts quantities according to equation (8) admits $p(\theta) = 0$ as a Markov Perfect Equilibrium for all θ.*

Proof: Postulate that $p(\theta) = 0$, $\forall\theta \in [0, \infty)$. Note from equation (1) that, when $p = 0$ and $\lambda_3 > 1$, $u'(C(\theta)/A(\theta)) = 0$. Applying the general solution (6) to the price of stablecoin $p(\theta)$ with both $r^c = 0$ and $u'(C(\theta)/A(\theta)) = 0$, we find the two consistent constants $c_1 = c_2 = 0$, which validates our postulate and completes the proof. □

The intuition behind this result is straightforward. Since the value of holding the stablecoin depends on its stability, there is no incentive for an individual investor to deviate from a zero valuation for the stablecoin. The second culprit of algorithmic protocols lies in limitations to the feasibility of their quantity adjustment policy. As the protocol directly pays out the proceeds of newly minted stablecoins, the ability of the system to withdraw stablecoins in reaction to a shock is limited to the size of its franchise value. Note that this is equal to the discount growth rate in the demand for the stablecoin. In this respect, an algorithmic stablecoin protocol resembles a Ponzi scheme. For the system to retain its ability to remove quantities when needed and stabilize prices, it needs to keep growing at an exponential rate.

Proposition 4 (Limitations to Quantity Control and Price Instability in Algorithmic Protocols). *Any algorithmic protocol—that is, $A^\ell(\theta) = B(\theta) = B^\ell(\theta) = 0\ \forall\theta$—that adjusts quantities according to equation (8) admits $p(\theta) = 1$ as a Markov Perfect Equilibrium for all $\theta \in [0, 1]$. For $\theta > 1$, the valuation $p(\theta) = 1$ is not a Markov Perfect Equilibrium.*

Proof: See Appendix A.

[Figure 7 about here]

Figure 7 illustrates the pricing of algorithmic stablecoins for the stable equilibrium. When θ is between 0 and 1, the price of the stablecoin is pegged to one due to continuous quantity adjustments. Once this threshold is reached, no adjustment in quantity is possible anymore and the price of the stablecoin becomes a negative function of θ. As the demand for the stablecoin keeps falling while quantities remain constant, prices have to drop. At some point, the stablecoin price crosses the tolerance for deviation from parity λ_3 and falls to zero. Note that, as introduced in Proposition 3, $p(\theta) = 0$ is always an equilibrium.

7 Decentralized Protocols

[Figure 8 about here]

In this section, we adapt our general model to account for *decentralized* stablecoin protocols. Such protocols—with Dai as its most prominent example—allow for the decentralized creation of new stablecoins by anyone with enough collateral. To do so, individual investors have to lock some collateral asset in a smart contract generated by the protocol—a *vault*—and can issue some stablecoins against it. The protocol only allows issuing a quantity of stablecoins that is strictly below the value of collateral assets to allow for some over-collateralization to absorb pricing volatility. Once the stablecoins are sold to outside investors, the vault represents for its owner a leverage position in the collateral asset. Moreover, the newly issued stablecoins are not tied to a particular vault and are fully indistinguishable from other stablecoins—i.e., decentralized stablecoins are perfectly fungible. Vault owners can unlock their collateral assets by repurchasing and "burning" enough stablecoins to liquidate the vault. The system's stability, therefore, relies on providing the right set of incentives for individual investors to adopt prudent risk management practices and not to over-extend the supply of stablecoins. As in the centralized case, the protocol also issues governance tokens with voting rights on the system's key parameters and claims to the system's seigniorage revenues. Furthermore, governance token investors may decide on an additional on-ledger currency buffer at the system's level to provide additional safety to the stablecoin. Figure 8 provides a consolidated balance sheet representation for decentralized stablecoins.

7.1 Environment

In a decentralized crypto-collateralized protocol, the system uses an on-ledger reserve buffer B^{ℓ} to cushion sharp movements in the collateral asset value A. Individual vaults indexed

$i \in [0,1]$ are created using collateral value A^i in exchange for a quantity C^i of stablecoins with price p. When the loan is repaid by the vault owner, the stablecoin is "burned" and removed from the supply. As there is no heterogeneity across infinitesimal vault owners, we consider the asset and debt value of an individual representative vault: $A = \int_0^1 A^i di$ and $C = \int_0^1 C^i di$ with collateralization ratio $\eta = A/C$. As for centralized crypto-collateralized protocols, the system relies on over-collateralization. The value of the asset stored in the vault should be above the threshold $\underline{\eta}$. When $A < \eta_\ell C$, i.e. when $\eta < \underline{\eta}$, the vault is liquidated. In such a case, the vault owner receives the asset at its spot price discounted for a loss due to fire-sale externalities. The distribution of losses at liquidation is detailed below.

Fees accrue through time between the different investors according to predetermined parameters. Stablecoin holders receive the convenience yield $L = u'(\eta)C$ as well as some interest rate on stablecoins r^c, which is paid by the governance system. Moreover, besides paying this interest to stablecoin holders, the governance system charges vault owners in two ways. First, vault owners pay the stability fee—denoted r^s—to the governance system. As a result, the net spread earned by the governance token is $(r^s - r^c)C$. Second, at liquidation, the vault owner pays a liquidation fee in proportion to the quantity of stablecoin issued ℓC.

As for centralized crypto-collateralized protocols, the scale invariance of the problem allows to solve for Markov perfect equilibria in a single state variable: the aggregated collateralization ratio $\eta_t = A_t/C_t$. Being solution of the PDE (5), the general solution has the same form as equation (6). Hence, we normalize our variables by the outstanding quantity of stablecoin C_t. The reserve buffer is $b(\eta) = B^\ell/C$; the franchise value created by the convenience yields is $u'(\eta)/r$; stablecoin price is $p(\eta, b)$, while governance tokens have value $g(\eta, b)$. Finally, discounting the system value $s(\eta, b)$ by the price of debt $p(\eta, b)$ and system equity $g(\eta, b)$, the aggregate equity in individual votes is denoted $e(\eta, b)$.

7.2 Pricing Equations for Collateralized Vaults

Vault owners store the collateral asset above the liquidation threshold $\underline{\eta}$. If the value of the asset falls below this limit, the vault is liquidated, and the vault owner receives the residual value of the asset, discounted for two costs: (i) a factor α representing the fire-sale externality and (ii) an additive liquidation fee ℓ imposed by the system. Hence, the boundary value of the equity is $e(\underline{\eta}) = (1-\alpha)\underline{\eta} - \ell$.

Stablecoin holders benefit from the convenience yield as long as the liquidation does not occur. As in previous sections, the franchise value for the stablecoin is $(r^c + L)/r$ at the limit where $\eta \to \infty$. However, since some of the collateral asset may remain with the vault owner at liquidation, the stablecoin holder receives only a value $\underline{p}_\ell$—a discount from the face value—and $\underline{p}_\ell = (1-\delta(b,\alpha))(r^c+L)/r$. To remain as general as possible, we allow for any possible relationship between the change in stablecoin price δ and the fire-sale externality α. Applying our general pricing formula (6), we find the pricing equation for stablecoins:

$$p(\eta, b) = \frac{r^c + L}{r}\left(1 - \left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}\right) + (1-\delta(b,\alpha))\frac{r^c+L}{r}\left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}. \tag{9}$$

The protocol aims at preserving the value of the stablecoin at liquidation. Consequently, it uses both the buffer b and the liquidation fees ℓ to repay stablecoin holders as close as possible to face value. Consider an example in which $\delta(b,\alpha) = 0$ when $b > \kappa_\delta \alpha (r^c + L)/r$. As long as there is enough buffer to cover the loss, the governance system prevents any depreciation in stablecoin value. As a result, governance token valuation is:

$$g(\eta, b) = b + \frac{r^s - r^c}{r}\left(1 - \left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}\right) + \left(\ell - (1-\delta(b,\alpha))\frac{r^c+L}{r}\right)\left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}. \tag{10}$$

Hence, we see that as long the value $g(\eta, b)$ is positive, the higher the buffer b, the lower the loss in value for the stablecoin $\delta(b,\alpha)$. We then find the total value of the system:

$$s(\eta, b) = \underbrace{\eta + b}_{\text{Assets value}} + \underbrace{\frac{L}{r}\left(1 - \left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}\right)}_{\text{Franchise value}} - \underbrace{\alpha\eta\left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}}_{\text{liquidation cost}}.$$

From the previous set of equations, we can derive the vault equity value as:

$$\begin{aligned} e(\eta, b) = e(\eta) &= s(\eta, b) - g(\eta, b) - p(\eta, b) \\ &= (\eta - \frac{r^s}{r})\left(1 - \left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}\right) + \left((1-\alpha)\underline{\eta} - \ell\right)\left(\frac{\eta}{\underline{\eta}}\right)^{-\nu}, \end{aligned}$$

where the value of the asset η is discounted at liquidation as explained above. Note that the equity value of the vault does *not* depend on either the loss of stablecoin value at liquidation $\delta(b, \alpha)$ nor the value of the buffer b. This outcome is explained by our assumptions regarding the distribution of losses: the system governance control the buffer to cover for the losses on the stablecoin price. These two facts have important implications for vault leverage choices.

7.3 Ratchet effect

As in previous sections, the pricing of the system echoes the standard model by (Leland, 1994), where the convenience yield provides incentives for the agents to increase leverage. The difference between these protocols and the centralized system is the infinitesimal nature of the individual vault owners. In particular, "shareholders" do not internalize their impact on the aggregate price of the stablecoin $p(\eta)$ when taking leverage decision, which might attenuate the leverage ratchet effect of Admati et al. (2018). Nevertheless, we show that the effect fully holds in this context, provided that the convenience yield does not depend on leverage, i.e. on $\eta = A/C$. Thus, $L = u'(\eta) = \lambda_1$ and $\lambda_2 = 0$: this assumption relies on the fact that individual vaults are infinitesimal. In such context, equity holders do not internalize their leverage decision on the convenience yield of the system as a whole.

Proposition 5 (Ratchet effect in decentralized protocols). *Upon receiving the opportunity, vault owners never approve a reduction in the system's leverage from $1/\eta$ to $1/\eta^\star$ with $\eta^* > \eta$.*

Proof: See Appendix A.

Reducing leverage decreases the convenience yield received on the total amount of debt. Despite decreasing the probability of liquidation from $(\eta/\underline{\eta})^{-\nu}$ to $(\eta^\star/\underline{\eta})^{-\nu} > (\eta/\underline{\eta})^{-\nu}$, the previous mechanism dominates. This ratchet effect is increasing in stability fees r^s—reducing the gain for equity holders—and decreasing in the cost at liquidation ℓ—amplifying the loss caused by a change in the probability of liquidation.

Since it is never profitable to reduce leverage, vault investor do not prevent the collateralization ratio η to fall to the liquidation threshold $\underline{\eta}$, causing a potential loss of the coin holders. This result differs from the one in a centralized system illustrated in Figure 4 and for which the inaction region in which ratchet effect applies is bounded below by $\eta_r > \underline{\eta}$.

7.4 Stability

As earlier, when the collateral value is large, the face value and hence the interest rate paid on stablecoins have to be consistent with parity, yielding :

$$r^c - r = u'(\eta).$$

This condition is not sufficient to keep the stablecoin stable as the leverage Ratchet effect prevents any increase in aggregate collateralization.

Proposition 6 (A decentralized system is less stable than centralized systems). *Consider any arbitrary fundamental shock on stablecoin value at liquidation* $\delta = \delta(b, \alpha)$*, then the decentralized system does not prevent the decline in the price* $p(\eta, b) < 1$

As a consequence, the system governance needs to set the parameters to prevent the decline in price $\delta(\alpha, b)$. This could be achieved by several levers, as can be viewed in 10: (i) an increase in liquidation fee ℓ, (ii) an increase in the stability fee, widening the spread $r^s - r^c$, and (iii) an increase in the reserve buffer b.

For the remainder of this section, we specialize the model further and assume that profits accruing to governance tokens are fully assigned to the reserve buffer b. Moreover, the whole buffer is used to prevent stablecoin depreciation, such that the loss in stablecoin value is:

$$\delta(b, \alpha) = \begin{cases} \kappa_\delta \alpha & \text{if } b = 0 \\ 0 & \text{if } b > \kappa_\delta \alpha (r^c + L)/r. \end{cases} \tag{11}$$

Keeping track of the dynamics of this state variable we obtain the process :

$$db_t = (r^s - r^c)dt + (\ell - \kappa_\delta \alpha (r^c + L)/r) dN_t(\lambda(\eta_t)).$$

In the previous equation, the drift is simply the period return, while the overall process is stochastic due to the regular occurrence of liquidation episodes. The process $dN_t(\lambda)$ is a Poisson process of intensity λ, which here depends on the collateralization of vaults η.

Proposition 7 (With active use of reserve buffers, the decentralized system can be more stable than the centralized system). *In the case where convenience yield is constant in* η*, such that* $L = u'(\eta) = \lambda_1$ *and* $\lambda_2 = 0$*, and consider the fundamental shock on stablecoin value at liquidation* $\delta = \delta(b, \alpha)$ *as show in equation 11, then the decentralized* *system can prevent the decline in the price $p(\eta, b) < 1$, if the risk management parameters of the system ℓ, r^s, r^c are such that:*

$$\mathcal{F}_\ell(\ell - \kappa_\delta \alpha \frac{r^c + L}{r}) + (r^s - r^c) > 0,$$

where

$$\mathcal{F}_\ell = (\mu - \frac{\sigma^2}{2}) \underline{\eta}^{-\frac{\sigma^2}{2}}$$

is the stationary flow of liquidation when $\eta = \underline{\eta}$.

Proof: See Appendix A.

Intuitively, the higher the drift of the diffusion μ and the lower the volatility σ^2, the lower the probability of liquidation when the collateral value is close to $\underline{\eta}$. In this case, the system can increase its reserve buffers by charging the stability spread $r^s - r^c$ for every smart contract or a liquidation fee ℓ covering the externality at liquidation.

8 Conclusion

In this work, we propose a general model of stablecoins and examine the merit and vulnerability of various stabilization mechanisms. Our analysis points out that stablecoin protocols share some—but not all—features with conventional financial institutions such as mutual funds, banks, and central banks. In particular, collateralization and liquidation covenants play a crucial role in the stabilization of crypto-collateralized protocols. We demonstrate that these schemes are highly dependent on the market liquidity of their collateral assets and are vulnerable to fire-sales spirals of the type observed during the 2008 financial crisis. In contrast, uncollateralized algorithmic schemes rely on irredeemable stablecoins and quantity adjustments with alleged inspiration from central banks. As for a central bank, we show that issuing irredeemable liabilities does not dispense from holding tangible assets. Otherwise, there is always a limit to how many stablecoins can be withdrawn when facing a negative demand shock, and the scheme loses its control over prices. Overall, our work has practical implications for the design and regulation of future stablecoins. In particular, we point to collateralization, automatization, and decentralization as essential stabilization tools.

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Appendices

A Omitted Derivations and Proofs

Proof of Proposition 2 We write the net gain in governance token in reducing leverage from $1/\eta$ to $1/\eta^\star$ per unit of wealth A as:

$$\mathcal{G}(1/\eta, 1/\eta^\star) = g(\eta^\star) - g(\eta) - p(\eta^\star)\big(1/\eta - 1/\eta^\star\big)$$

which can be rewritten as:

$$\begin{aligned}\mathcal{G}(1/\eta, 1/\eta^\star) =& \frac{u'(\eta^\star)}{r\eta^\star} - \frac{u'(\eta)}{r\eta} - \left(\frac{u'(\eta^\star)}{r\eta^\star} - \frac{u'(\eta)}{r\eta}\right)\left(\frac{\eta}{\underline{\eta}}\right)^{-\nu} \\ & - \big((1-\alpha)\underline{\eta}\big)\left(\frac{\eta^\star}{\underline{\eta}}\right)^{-\nu}\left(\frac{1}{\eta} - \frac{1}{\eta^\star}\right)\end{aligned}$$

Taking into account the condition that $\lambda_2 < 1$ and applying the functional form in equation (1), we have that:

$$\mathcal{G}(1/\eta, 1/\eta^\star) = \frac{\lambda_1\left[\lambda_2((1/\eta)^2 - (1/\eta^\star)^2) - ((1/\eta) - (1/\eta^\star))\right]}{r} - \big((1-\alpha)\underline{\eta}\big)\left(\frac{\eta^\star}{\underline{\eta}}\right)^{-\nu}\left(\frac{1}{\eta} - \frac{1}{\eta^\star}\right)$$

Note that for governance token holders to approve a leverage reduction, they need to benefit from it, $\mathcal{G}(1/\eta, 1/\eta^\star) > 0$, which gives equation (2) and completes the proof. $\square$

Proof of Proposition 5 Note that infinitesimal vault owners do not internalize the impact of their leverage on the convenience yields and, hence, treat it as a constant L. With $C^\star < C$, a reduction in debt —at price $p(C,A)$ since there is also no internalization of the effects on prices—gives the gain for vault equity owners:

$$\begin{aligned}
\mathcal{G}(A,C,C^\star) &= \tilde{E}(A,C) - \tilde{E}(A,C^\star) - p_c(A,C)\big(C - C^\star\big) \\
&= (A + \frac{L-r^s}{r}C^\star) + (\frac{A}{\eta C^\star})^{-\nu}\big((1-\alpha)A - \ell C^\star - \frac{L-r^s}{r}C^\star\big) \\
&\quad - (A + \frac{L-r^s}{r}C) - (\frac{A}{\eta C})^{-\nu}\big((1-\alpha)A - \ell C - \frac{L-r^s}{r}C\big) \\
&\quad - \underbrace{\frac{r^c+L}{r}\Big(1-\delta(\frac{A}{\eta C})^{-\nu}\Big)}_{p(A,C)}(C-C^\star) \\
&= -\frac{L-r^s}{r}\underbrace{(C-C^\star)}_{>0} + \Big[(\frac{A}{\eta C^\star})^{-\nu} - (\frac{A}{\eta C})^{-\nu}\Big](1-\alpha)A \\
&\quad + \Big[(\frac{A}{\eta C})^{-\nu}C - (\frac{A}{\eta C^\star})^{-\nu}C^\star\Big](\ell + \frac{L-r^s}{r}) - \frac{r^c+L}{r}\Big(1-\delta(\frac{A}{\eta C})^{-\nu}\Big)(C-C^\star) \\
\mathcal{G}(A,C,C^\star) &= -\frac{L-r^s}{r}\Big[\underbrace{C\big(1-(\frac{A}{\eta C})^{-\nu}\big) - C^\star\big(1-(\frac{A}{\eta C^\star})^{-\nu}\big)}_{>0}\Big] \\
&\quad + \Big[\underbrace{(\frac{A}{\eta C^\star})^{-\nu} - (\frac{A}{\eta C})^{-\nu}}_{=v_1<0}\Big](1-\alpha)A + \Big[\underbrace{(\frac{A}{\eta C})^{-\nu}C - (\frac{A}{\eta C^\star})^{-\nu}C^\star}_{=v_2>0 \text{ but } <-v_1}\Big]\ell \\
&\quad - \frac{r^c+L}{r}\Big(1-\delta(\frac{A}{\eta C})^{-\nu}\Big)(C-C^\star)
\end{aligned}$$

This is negative at the conditions that: (i) $(1-\alpha)A - \ell C > 0$, indeed if the liquidation fees are too large to overcome any potential gain the vault owner would obtain at liquidation, the change in probability of default of the system is too important for the leverage decision, incentivizing to reduce C to $C^\star$, and (ii) $\lambda < 1$, i.e. if the elasticity of default probability to leverage is not too high, the quantity $C\Big(1-(\frac{A}{\eta C})^{-\nu}\Big) = \frac{1}{\eta}(1-\Big(\frac{\eta}{\underline{\eta}}\Big)^{-\nu})$ is increasing in C, the gain in terms of liquidity, by a factor $\frac{L-r^s}{r}$, is stronger than the change in the

Normalized by A to obtain a formula in η: reduction in leverage from low η to high $\eta^\star$

(more collateralization)

$$\mathcal{G}(\eta,\eta^\star) = \tilde{E}(A,C) - \tilde{E}(A,C^\star) - p_c(A)\big(C - C^\star\big) \tag{12}$$

$$= -\frac{L-r^s}{r}\Big[\underbrace{\frac{1}{\eta}\big(1-\Big(\frac{\eta}{\underline{\eta}}\Big)^{-\nu}\big) - \frac{1}{\eta^\star}\big(1-\Big(\frac{\eta^\star}{\underline{\eta}}\Big)^{-\nu}\big)}_{>0}\Big] + \underbrace{\Big(\Big(\frac{\eta^\star}{\underline{\eta}}\Big)^{-\nu} - \Big(\frac{\eta}{\underline{\eta}}\Big)^{-\nu}\Big)}_{>0}(1-\alpha) \tag{13}$$

$$+ \underbrace{\underline{\eta}^\lambda\Big[\eta^{-\lambda+1} - \eta^{\star\,-\lambda+1}\Big]\ell}_{>0} - \frac{r^c+L}{r}\Big(1-\delta\big(\frac{A}{\underline{\eta}C}\big)^{-\nu}\Big)\big(C-C^\star\big) \tag{14}$$

□

Proof of Proposition 7 We start with a setting where the governance system possess a quantity b of reserve buffer, such that when an infinitesimal vault is liquidated, a new one is created a collateralization $\underline{\eta}$.

Let us consider the process :

$$d\eta_t/\eta_t = \mu dt + \sigma dZ_t$$

subject to the boundary condition $\eta \leq \underline{\eta}$ The stationary distribution of this reflected geometric Brownian motion follows the Kolmogorov forward equation

The distribution of η_t is given by $h(t,\eta)$ that solve the Kolmogorov forward equation

$$\partial_t h(t,\eta) = -\frac{d}{d\eta}[\mu\eta h(t,\eta)] + \frac{d}{d\eta^2}\big[\frac{\sigma^2}{2}\eta^2 h(t,\eta)\big]$$

If $\partial_t h(t,\eta) = 0$, this is equivalent to an equation of the type (for $\eta \equiv x$ here, to alleviate the notations)

$$h(x) + xh'(x) = \frac{\sigma^2}{2\mu}\Big(2h(x) + 4xh'(x) + x^2h''(x)\Big)$$

Which gives the standard solution as the following: (here with the power are specific to this problem:)

$$h(x) = c_1\frac{1}{x} + c_2 x^{-\frac{\sigma^2-\mu}{\sigma^2/2}}$$

However, the boundary condition at the threshold is specific to the presence of reflecting barrier:

$$\mu x h(x) = \frac{\sigma^2}{2}\frac{d}{dx}[x^2 h(x)]$$

Intuitively, the drift out has to be compensated by the variance (or more specifically here, the variance out has to be compensated by the drift in). This yields $c_1 = 0$, which could also have been seen by definition of density (you can't have an inverse function in density as is doesn't sum to one). So in this case the constant is simply a renormalization constant, making the mass some to one:

$$\int_{\underline{\eta}}^{\infty} c_2 x^{-\frac{\sigma^2-\mu}{\sigma^2/2}}\,dx = 1 \qquad \Rightarrow \qquad c_2 = \frac{\sigma^2/2}{\mu-\sigma^2/2}\underline{\eta}^{-\frac{\mu-\sigma^2/2}{\sigma^2/2}}$$

In particular, this restrict the range of parameter to be: $\sigma^2/2 < \mu$, for this density to be positive (the drift is higher than the variance term), but from before we also had: $\mu < \sigma^2$, for the density to be decreasing (for the particle not to drift to far up the distribution).

As a result the distribution for $\eta \equiv x$ (readopting the original notation) is

$$h(\eta) = \frac{\sigma^2/2}{\mu-\sigma^2/2}\underline{\eta}^{-\overbrace{\frac{\mu-\sigma^2/2}{\sigma^2/2}}^{=+\frac{\sigma^2-\mu}{\sigma^2/2}-1}}\eta^{-\frac{\sigma^2-\mu}{\sigma^2/2}} = \frac{\sigma^2/2}{\mu-\sigma^2/2}\underline{\eta}^{-1}\left(\frac{\eta}{\underline{\eta}}\right)^{-\frac{\sigma^2-\mu}{\sigma^2/2}}$$

Moreover, what we wanted originally is the cumulative decrease out of the set (for η_t and η_t, which have the same distribution $h(\cdot)$ here).

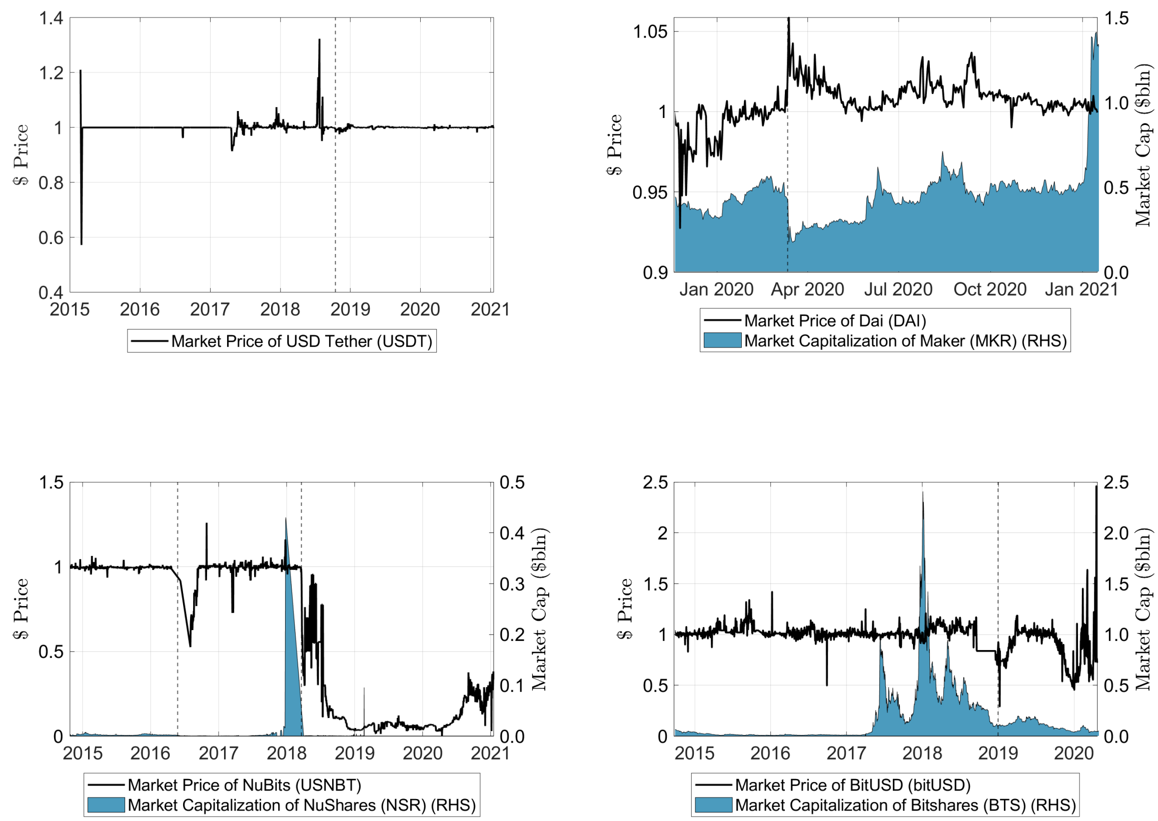
This is given by $\mathcal{F}_{out,d} = \frac{\sigma^2}{2}h(\underline{\eta})$, the variance term that makes particles push through the barrier. In our case, thanks to the calculation above we get:

$$\mathcal{F}_{out,d} = \frac{\sigma^2}{2}h(\underline{\eta}) = \frac{\sigma^2}{2}\frac{\mu-\sigma^2/2}{\sigma^2/2}\underline{\eta}^{-\frac{\sigma^2}{2}} = (\mu-\sigma^2/2)\underline{\eta}^{-\frac{\sigma^2}{2}}$$

This is the average flow of η_t out of the set. □

B Figures

Figure 1: Stablecoin Price Series This figure illustrates the price history of four stablecoin systems. USD Tether is a fiat-based, custodial stablecoin; Dai, NuBits and bitUSD are non-custodial systems. They are, respectively, on-chain collateralized with exogenous collateral, algorithmic with implicit collateral and on-chain with endogenous collateral.
Source: Coingecko.com



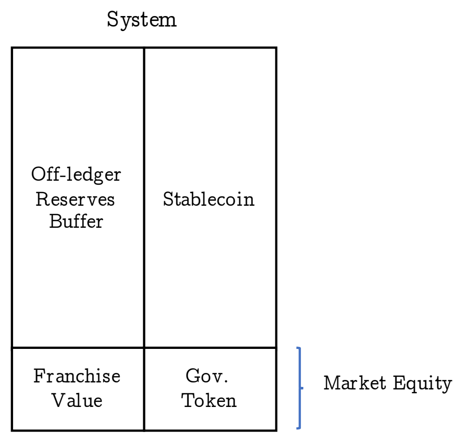


Figure 2: Balance Sheet of a Reserve Fund Protocol

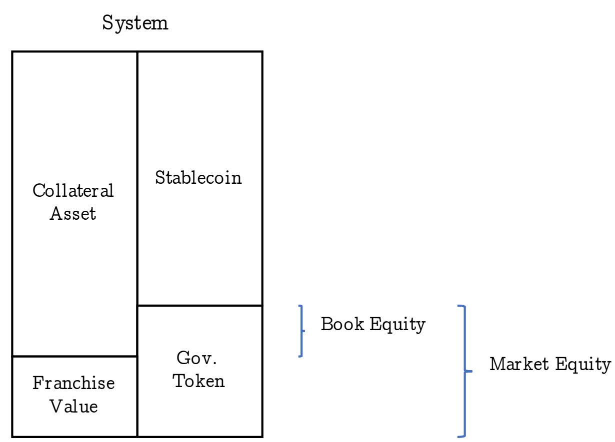
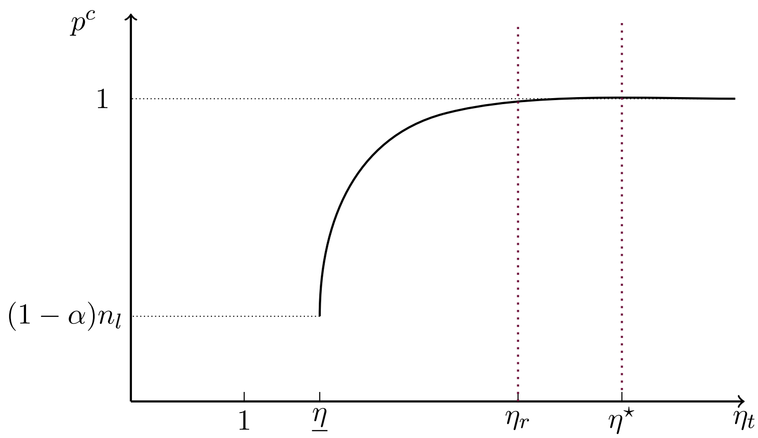
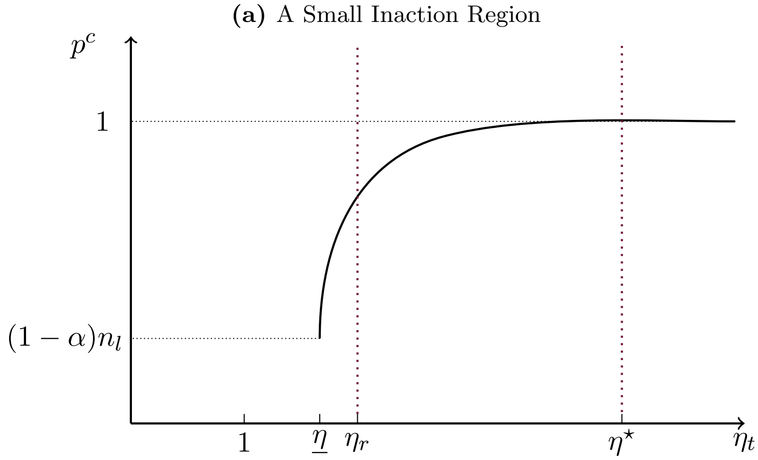


Figure 3: Balance Sheet of a Crypto-collateralized Protocol



(a) A Small Inaction Region



(b) A Large Inaction Region

Figure 4: Stablecoin Price Dynamics

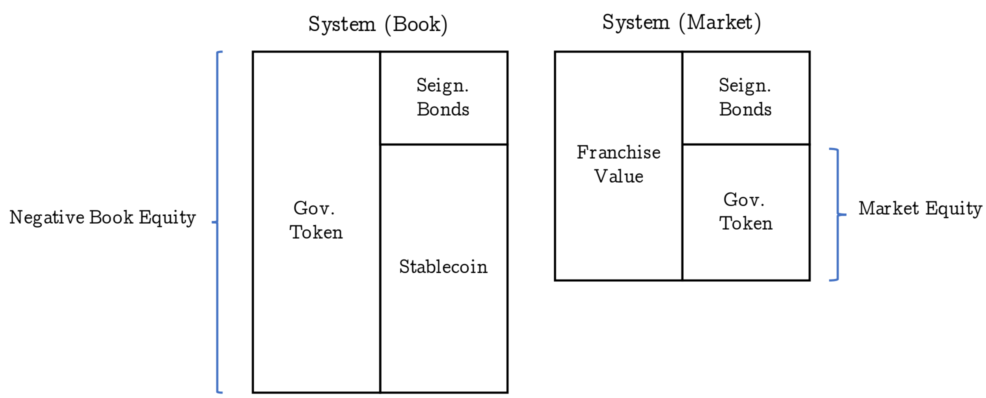


Figure 5: Balance Sheets of an Uncollateralized Protocol

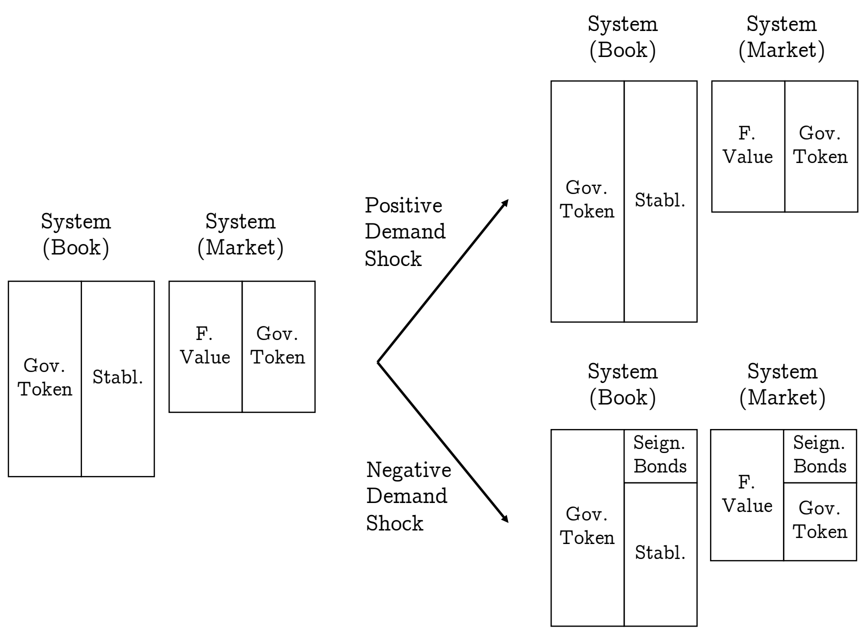


Figure 6: Uncollateralized Protocol Balance Sheet Adjustments to Demand Shocks

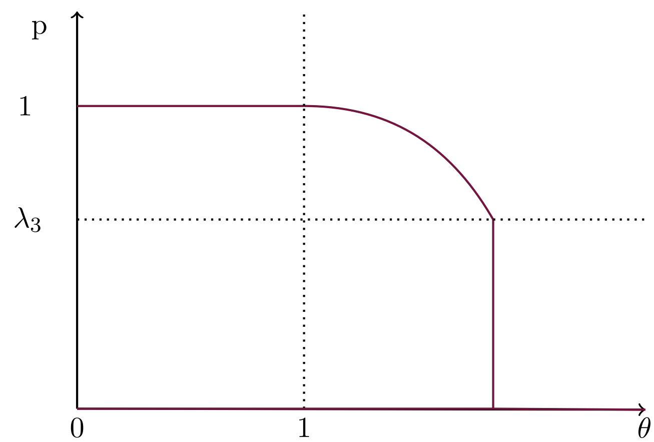


Figure 7: Uncollateralized Stablecoin Price Dynamics

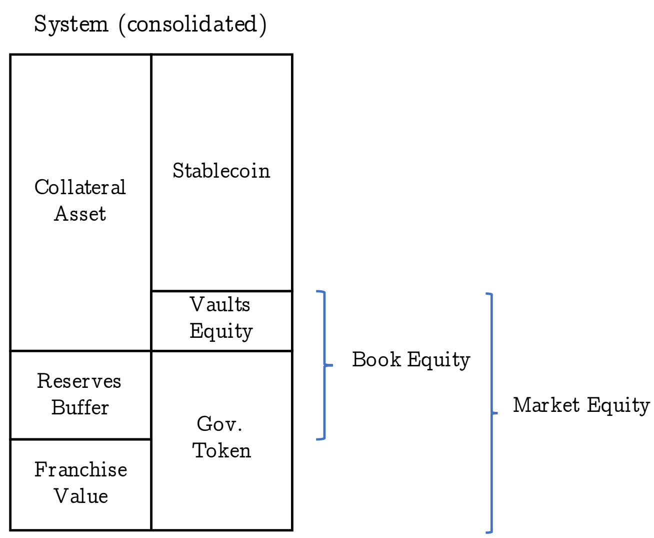


Figure 8: Consolidated Balance Sheets of a Decentralized Protocol